DOCUMENT RESUME

ED 130 827 SE 020 621

AUTHOR Rexroat, Melvin E.
TITLE Math Objectives and Utilization of State Math Curriculum Guide. Module CMM:006:03.
INSTITUTION Idaho State Univ., Pocatello.
PUB DATE Sep 72
NOTE 67p.; For related Modules, see SE 020 618-624; Not available in hard copy due to marginal legibility of original document

EDRS PRICE MF-$0.83 Plus Postage. HC Not Available from EDRS.
DESCRIPTORS *Behavioral Objectives; Curriculum; *Elementary School Mathematics; Elementary School Teachers; Higher Education; Instruction; *Instructional Materials; Mathematics Education; Methods Courses; *Performance Based Teacher Education; *Preservice Education; Teacher Education

ABSTRACT

The third in a sequence of modules for preservice elementary school mathematics teachers, this module covers behavioral objectives and the use of state mathematics curriculum guides. Objectives for the module are specified; prerequisites are listed; and pre- and post-assessment and remediation activities are given. Ten instructional activities to be completed by the preservice teachers are listed. Nine reading handouts are included with the module. (DT)

****
* Documents acquired by ERIC include many informal unpublished *
* materials not available from other sources. ERIC makes every effort *
* to obtain the best copy available. Nevertheless, items of marginal *
* reproducibility are often encountered and this affects the quality *
* of the microfiche and hardcopy reproductions ERIC makes available *
* via the ERIC Document Reproduction Service (EDRS). EDRS is not *
* responsible for the quality of the original document. Reproductions *
* supplied by EDRS are the best that can be made from the original. *
****

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

BEST COPY AVAILABLE

ED130827

Objectives:

Upon completion of Module study, the Intern will be able to:

1. (KNOWLEDGE) Identify the following information contained in the State Math Curriculum Guide:

   a. The role of Behavioral Objectives in math Learning.
   b. The role of the Teacher in math Learning.
   c. The role of problem solving in math Learning.
   d. The 5 major topics utilized in organizing the math concept continuum.

2. (Knowledge and Performance) Using the State Math Guide, IDENTIFY the "procedures for" and DEMONSTRATE "applications of" the activity called "What's My Rule?" with a group of Learners AND the same with at least one other technique or material listed in the Addenda section of the State Math Guide.

3. (Knowledge and Performance) DEMONSTRATE a method for locating particular sets of behavioral objectives in the State Math Guide utilizing the Scope Sequence charts, pp. xxi-xxvi of the State Math Guide.

4. (Knowledge) STATE several learner behaviors, common to math, which represent at least 3 different levels or categories of the Cognitive Domain and the Affective Domain.

5. (Knowledge) CLASSIFY and WRITE objectives of math as Cognitive, Affective, or Psychomotor.

6. (Knowledge) CONSTRUCT and APPLY with learners instruments and/or procedures for assessing learners' behaviors in math for at least 3 out of 6 levels of the Cognitive Domain, using a Behavior-Content Grid. (SEE: Sund and Picard Handout: "The Behavior-Content Grid.")

7. (Performance) DETERMINE the distribution of items on a given assessment among the categories or levels of the Cognitive Domain, using a Behavior-Content Grid. (SEE: Handout: Behavior-Content Grid.)

8. (Performance and Consequence) Using page 7 of the Handout in Module CMM:006:01(ISU) entitled "Facilitating Inquiry in the Classroom," DESIGN, DEMONSTRATE with a group of learners, and EVALUATE using the Interaction Analysis Process, a "STRUCTURING LESSON" in math. (SEE: "Application of Structuring Lesson" sheets to be used with this objective.)

Pre-Requisites:

1. Modules: CMM:006:01(ISU) "Orientation to Math Methods and Materials"
   CMM:006:02(ISU) "Instuctional and Learning Modes in Math"

SE020 621

Pre-Requisites: (Cont'd)

2. Ability to IDENTIFY, CLASSIFY, and WRITE Behavioral Objectives (including the criteria of (a) Learner Terminal Behavior, (b) Conditions, and (c) Minimal Performance Level for at least 3 categories or levels of the Cognitive and Affective Domains. (If the Intern has successfully completed the Modules TTP:004:01(ISU) through TTP:004:07(ISU), then this Pre-Requisite is satisfied. If the above modules have not been completed satisfactorily the Intern must demonstrate the above requisites in one of the following ways:

   a. SCHEDULE and COMPLETE Modules TTP:004:01(ISU) through TTP:004:07(ISU). (These are self-instructional and would require about 1 hour each. Request from Instructor.)

   b. SCHEDULE and COMPLETE Module TTP:004:31(ISU) "WHY Behavioral Objectives." (This is a Tape-Flip Chart Program that is self-instructional. Request from Instructor.) and Module TTP:001:32(ISU) "Objectives Market Place Game." (This is a Programmed, self-instruction Package. Request from Instructor.) Each of these modules will take about 1 hour each to complete.

   c. Confer with the Instructor to orally and in writing demonstrate your ability to IDENTIFY, CLASSIFY, and WRITE Behavioral Objectives.

Pre-Assessments:

1. Objectives 1, 2 (Knowledge Portion), 3, 4, 5, and 6 (Knowledge Portion) may be demonstrated by completing the Pre-Assessment with a Minimum Performance Level of 90% correct.

2. Objectives 2 and 6 (Performance portions), 7, and 8 may be completed through Pre-Assessment since they involve Performance and Consequence Objectives, but ONLY IF THE INTERN Already possesses the Knowledges and SKILLS in the use of the activity called "What's My Rule?" (Objective 2), the Behavior-Content Grid (Objectives 6 and 7), and "STRUCTURING" and Interaction Analysis Process (Objective 8). If these Knowledges and Skills are not already possessed by the Intern, then these Objectives will require Module study prior to a Post-Assessment demonstration.

Instructional Activities: (NOTE: The following activities are options which the Intern may use for developing the Knowledges and Skills specified in the Objectives.)

1. READ:

   a. The Idaho State Math Curriculum Guide. (A Copy is available on loan from the TC OFFICE.)

   b. Handout: "IPI Math Continuum."

Instructional Activities: (Cont'd)

c. Handout: Walbesser: "Behavioral Objectives; a Cause Célebre."
d. Handout: Nichols: "Are Behavioral Objectives the Answer?"
e. Handout: Buffie: "Goal-Oriented Insruction in Math."
f. Handout: Briggs: "Scope and Sequence."
g. Handout: Edwards: "Mathematical Competencies and Skills Essential for Enlightened Citizens."
h. Handout: Sund and Picard: "Goals of Mathematics."
i. Handout: Sund and Picard: "OVERT and COVERT (Affective) Learner Behaviors in Math."
j. Handout: Sund and Picard: "Behavior-Content Grid."
k. (NOTE: Several Handuts included in Module TTP:004:03(ISU) would also prove helpful for original study or review of the Cognitive, Affective, and Psychomotor Domains. These Handouts include excellent writing helps:
   (1) KIBLER: "Examples of Infinities and Direct Objects--Cognitive Domain."
   (2) KIBLER: "Examples of Infinities and Direct Objects--Affective Domain."
   (3) KIBLER: "Psychomotor Domain."
   (4) GRONLUND: "General Instructional Objectives and Behavioral Terms--Cognitive, Affective, and Psychomotor Domains."
   (5) "A Condensed Version of the Cognitive Domain of the Taxonomy of Educational Objectives."
   (6) "A Condensed Version of the Affective Domain of the Taxonomy of Educational Objectives."
   (7) ESBENSEN: "Witing Instructional Objectives."
   (8) "Guidelines for writing Behavioral Objectives."

2. Explore Idaho Curriculum Guide in math (K-12). (A copy may be borrowed from TC Office, ISU.) In your exploration of the math Guide note the following:
   a. The role of Behavioral Objectives stated in the Philosophy of the Guide.
   b. The major topics and the color coding used for each.
   c. The elements or characteristics of the Behavioral Objectives included in the Guide.
   d. The definition of "problem solving," the nature of the behaviors involved in problem solving, and approaches to problem solving illustrated.
   e. The role of the teacher stated & implied by the math Guide.
   f. The meaning and use of the Scope and Sequence Charts pp. xix-xxvi.
   g. The Techniques and materials included in the Addenda section of the Guide:
      (1) "R & R Packaging machine" (place value. . .)
      (2) "The Board STRETCHERS" (multiplication, division; rational numbers and whole numbers).
      (3) "Function Machine" or "What's My Rule?" (many versions or variations covering patterns, testing ideas, and practicing using the "Games to use with section on Sets".)
      (4) "Suggested Laboratory Materials for use with Sets."
      (5) "Multiplication Table" (useful for multiplication & division and patterns. . .)
      (6) "Problems" (involving "Topology" concepts.)
      (7) "Number patterns"

Instructional Activities: (Cont'd)

(8) "Divisibility Rules"
(9) "A Rationale in Working with Signed Numbers" ("Postman Stories" model all operations. . .)
(10) "Matrikes," "Battleship," and "TIC TAC TOE" games.
(11) "Measurement and Estimation" ("Measurement Laboratory" approach. .)

3. In the Handout by Sund & Picard: "Goals of Mathematics," study the Math Behaviors identified in various goal statements and then compare an existing lesson plan or an observed lesson in math with those behaviors to determine what agreement the lesson has with the recommended behaviors in the Handout.

4. Using the Handout by Sund & Picard "Behavior-Content Grid" and both the Idaho Math Guide and the IPI Continuum, make a set of assessment instruments OR procedures by analyzing objectives within some topic to determine the levels of objectives on the Cognitive Domain. You will need to devise the "BEHAVIORS" to be analyzed by referring to the various levels of the cognitive Domain and the "CONTENT" will be selected from the objectives in the Idaho Math Guide and theIPI Continuum. Your assessments should include at least 3 levels of the Cognitive Domain.

5. Refer to Handouts and the State Math Guide or other sets of objectives and attempt to complete a chart of learner behaviors in math which represents the categories of the Cognitive Domain and Affective Domain. Identify both verbal and non-verbal learner behaviors for the various categories or levels of each Domain: (NOTE: more than one sub-category within each major category should be included.)

Cognitive Chart Example:

Learner Behaviors →

Cognitive Domain Category	Verbal	Non-Verbal
Knowledge: (1.00) 1.11 Knowledge of terminology	Learner will identify the meaning of a set by stating that "a set is a collection.."	
1.12 Knowledge of specific facts		Learner will recognize the properties of a sub-set by placing all red objects in one stack and all blue objects in another stack...
ETC.		

Instructional Activities: (Cont'd)

Affective Chart Example:

Affective Domain Category	Learner Behavior	
	Verbal	Non-Verbal
Receiving (1.00) 1.20 Willingness to Receive	Learner will select three examples out of five listed when asked: "Can you find any other examples of problems which are like this n + 3= 8	Learner will combine into different stacks all objects of the same size or color when asked: "Can you place all of the objects of the same color or size in different boxes?"
ETC.		

6. Refer to sources concerning Cognitive, Affective, and Psychomotor Domains and various sources of math objectives (your own lesson plans, books, State Math Guide, IPI Continuum, etc.). List several sample objectives and then clarify them according to which Domain they seem to primarily represent:

Examples:

Objectives	Cognitive Domain	Affective Domain	Psychomotor Domain
1. Learner is asked to demonstrate by reading sets on a numbers line that 3 X 4 = 12 and 4 X 3 + 12 (p.41 State Math Guide)	X		
2. The Learner "volunteers" an explanation of 3 X 4 = 12 and 4 X 3 = 12 by drawing jumps on a number line as follows: 0 1 2 3 4 5 6 7 8 9 10 11 12		X	
3. The Learner writes the numerals 1 thru 10 without error.			X
ETC,			

Instructional Activities: (Cont'd)

7. Using the State Math Guide as a source for assessment items (SEE: The middle column in the Guide entitled "Behavioral objectives") and using the Behavior-Content Grid (blank copies attached), select at random a sample of at least 20 objectives and DETERMINE the distribution of the items among the levels of the Cognitive Domain. You will need to refer to Handouts to identify the Cognitive levels along the top of the CHART and the Learner Behaviors or Content along the left side of the CHART. (NOTE: It may be helpful to refer to the KIBLER and GRONLUND Handouts (module TTP:004:03(ISU) in determining the Levels of the Cognitive Domain of each objective. (NOTE: If you have access to an assessment which has been given or will be given to learners, you could analyze the items it includes on the Behavior-Content Grid instead of drawing a sample of objectives from the State Math Guide.)

8. Using a Behavior-Content Grid which identifies at least 3 levels of the Cognitive Domain, build an assessment instrument or procedure (could be verbal, non-verbal, or manipulative) which covers a short lesson or topic of instruction in math. In order to use the Behavior-Content Grid for building assessments, you will need to generally follow these steps:

   a. Label the 6 levels of the Cognitive Domain across the top of the Grid. Example?

Assessment Items for Lesson or topic ↓	Knowledge of Definitions and Terms (1)	Skills and Techniques in manipulation & computation (2)	Translation & Interpretation (3)	Analysis: select relevant & Irrelevant Data (4)	Etc. (5)
Etc.					

   b. Within the lesson or topic chosen, write sets of assessment items which represent a sampling of each level, say 2 in the (1) level, 3 in the (2) level, 2 in the (3) level, and so on.

   c. Try to include at least 3 levels of Objectives, more if possible. Now you will have built an assessment instrument or procedure which measures "high level" as well as "low level" cognitive Knowlege of Learners.

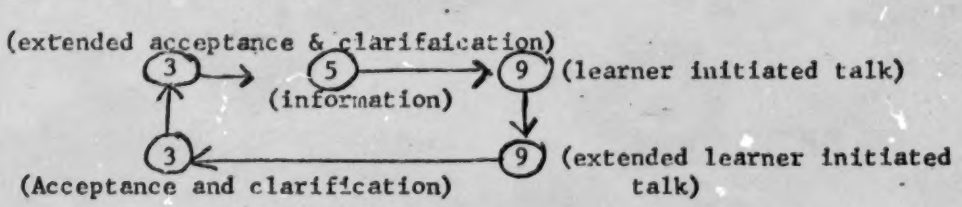
   d. Arrange your items or procedures on a sheet and give it to a group of learners. When the assessment is evaluated, determine how the learners were able to satisfactorily complete both low and high level cognitives. You might be surprised at the Power of Learners to demonstrate high order cognitives.

Instructional Activities: (Con'd)

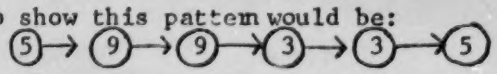
9. Refer to page 7 in the Handout in module CMM:006:03(ISU) entitled "Facilitating Inquiry in the Classroom." You will notice that this outline of a strategy calledSTRUCTURING displays a "conern with learning and practicing the Moves which ALLOW INQUIRY TO HAPPEN." Each move is described in 4 ways:

   TEACHER INTENT: (An unobservable purpose you want to fulfill.)
   TEACHER ACTION: (Something observable you do to fulfill your intent.)
   LEARNER ACTION: (Something observable you expect the learner to do in response to your action.)
   LEARNER EFFECT: (Some unobservable inner state of the learner that you hope to be the result of your action.)

   Your TASK is to DESIGN a "Structuring lesson" (Not a structured lesson) using any appropriate Topic in which you identify with the Teacher Intent and Teacher Action columns of the Page 7 CHART. You might find the activity on pp. 3 and 4 of the Buffie Handout "Goal-Oriented Instruction in Math" helpful. This lesson will require you to translate the page 7 CHART with your topic in mind. You should also anticipate what your possible examples of LEARNER ACTIONS will be. (Use the BLANK chart of page 7 which is attached to plan your lesson.) Then schedule a small group of learners and DEMONSTRATE the "Structuring moves." During the lesson, schedule the Team Leader to OBSERVE your demonstration and have him/her CODE YOUR LESSON USING FLANDERS INTERACTION ANALYSIS PROCESS OR Tape Record and CODE your own lesson to determine whether you carried out the Intent of the Structuring moves. (NOTE: The primary pattern of Interaction you will likely get in the MATRIX is:



   Another way to show this pattern would be:



   A blank Coding Sheet for Interaction Analysis is attached along with a blank MATRIX for recording the observations. (If you have not used the Interaction Analysis Process, then you will need to receive instruction from the Team Leader or Instructor PRIOR to attempting this analysis.)

10. DESIGN and CARRY OUT your own Instructional Activities related to the objectives in this module.

Post-Assessments:

1. Complete Post-Assessment with the Minimal Performance Level Indicated.

Post-Assessments: (Cont'd)

2. Attach to the Module Record Form the following as evidence of successful completion of objectives shown:

   Objective 2: A copy of the "What's My Rule?" activity you identified & demonstrated with a group of learners.

   Objective 6: A copy of the assessment instrument or a description of the assessment procedures which you DESIGNED and APPLIED with a group of learners using the Behavior-Content Grid.

   Objective 7: A copy of the analysis which you did on an assessment (or the sampling of objectives in the State Math Guide) using the Behavior-Content Grid.

   Objective 8: A copy of your plan for a "structuring Lesson," copies of the Coding Sheet and Matrix which you used to evaluate your lesson.

Remediation:

It may be necessary for the Intern to re-cycle some of their initial efforts with the Behavior-Content Grid and with the Interaction Analysis Process. The latter skill may require the scheduling of Team Leader and/or Instructor conferences or seminar. This may be the first application of the Interaction Analysis Process for Interns and Team Leaders since their exposure during Pre-Service.

MODULE RECORD FORM

Intern_____

Module No._____ Title:_____

Date Started:_____

Date Completed:_____

Record of Team Leader Conferences: (Team Leader please initial)

Dates	TL Initials

Record of Instructor Seminars: (Instructor please initial)

Dates	Inst. Initials

Written materials completed. (Please attach Pre- and Post-Assessments and written records on Instructional Activities.)

Send this form and attached materials to the Teacher Corps Office.

For TC Office use ONLY:

Included in _____course credit.

Date Module Satisfied_____Approval_____

REMARKS:

MODULE FIELD-TEST EVALUATION FORM

INTERN

MODULE # _____ TITLE _____

COURSE # _____ MODULE BEING STUDIED BY _____ (INTERN)

---

1. INDICATE WHICH CATEGORY YOU BELIEVE BEST DESCRIBES YOUR EVALUATION OF THIS MODULE: (CHECK ONE)

____A. KEEP-AS-IS
____B. DELETE ENTIRELY
____C. MODIFY BY DELETING PORTIONS (UNDER #2 IDENTIFY WHICH PORTIONS SHOULD BE DELETED.)
____D. MODIFY BY ADDING PORTIONS (UNDER #2 IDENTIFY WHICH PORTIONS SHOULD BE ADDED.)
____E. MODIFY BY RE-WRITING OR RE-DESIGNING PORTIONS (UNDER #3 IDENTIFY THE KINDS OF RE-WRITE OR RE-DESIGNING YOU VIEW NEEDED--BE SPECIFIC.)
____F. OTHER (SPECIFY UNDER #2).

2. SUGGESTIONS FOR MODIFICATIONS OF MODULES: (FROM #1--C, D, E, OR F ABOVE).

---

3. HOW DOES THIS MODULE SATISFY THE FOUR CRITERIA FOR A MODULE LISTED BELOW? (YOUR COMMENTS ARE WELCOMED).

	NOT AT ALL	LITTLE	SOME	VERY WELL
A. "FIELD-ORIENTED"--(UTILIZES KNOWLEDGES AND SKILLS IN APPLICATIONS WITH LEARNERS IN THLIR USUAL LEARNING ENVIRONMENTS IN THE SCHOOL AND COMMUNITY):	_____	_____	_____	_____
COMMENTS:				
B. "COMPETENCY-BASED"--(REQUIRE DEMONSTRATION BY INTERN OF CRITERION-REFERENCED COMPETENCIES IN KNOWLEDGES, PERFORMANCES, SKILLS, ATTITUDES, ETC.)	_____	_____	_____	_____
COMMENTS:				

MODULE FIELD-TEST EVALUATION FORM

INTERN

	NOT AT ALL	LITTLE	SOME	VERY WELL
C. "PERSONALIZING"--(ALLOWS INTERN TO CHOOSE FROM AMONG ALTERNATIVE INSTRUCTIONAL ACTIVITIES INCLUDING DESIGNING HIS/HER OWN, RELATED TO THE OBJECTIVES OR COMPETENCIES OF THE MODULE):	_____	_____	____	_____
COMMENTS				
D. "EMPHASIZE SATISFACTION OF PRODUCT OR CONSEQUENCE OBJECTIVES"--(EMPHASIS UPON LEARNER PERFORMANCE OUTCOMES--WHAT CHANGES TAKE PLACE IN CHILDREN AS A CONSEQUENCE OF THE INTERN'S INSTRUCTIONAL EFFORTS):	_____	_____	____	_____
COMMENTS				

TC OFFICE

DATE EVALUATED: _____

EVALUATORS NAME _____

CHECK ONE:
INTERN _____
TEAM LEADER _____
COOPERATING TEACHER _____
INSERVICE TEACHER _____
OTHER _____ (IDENTIFY)

MAIL OR SEND TO TC OFFICE I.S.U., POCATELLO

NUMERATION

Level A:

1. Identifies same, different; top, bottom; smaller, larger; smallest, largest.
2. Counts orally from 1 to 10.
3. Presented with numbers 1 to 10 in order, reads them orally left to right.
4. Counts orally from 1 to 10 objects by pointing to object and saying number.
5. Matches two equivalent sets of objects in a 1 to 1 relationship. Matches sets to 10.
6. Identifies the cardinal numbers of structured groups to 10.
7. Selects or constructs a set that contains as many objects as a given number.
8. Identifies the empty set or the set with zero numbers.
9. Matches two non-equivalent sets of 1 to 1 and indicates which has more or less.
10. Tells what number comes before or after a given number, or in-between two numbers, Numbers to 10.
11. Writes the numbers from 1 to 10.
12. Writes numbers 1 to 10 from left to right on an ordered set of pictures.

Level B:

1. Given number words for numbers zero to ten, reads words orally and matches words with numerals or structured groups.
2. Counts orally by 10's to 100 starting with tens only.
3. Counts orally by 1's to 100 in short sequences.
4. Presented with an ordered arrangement of numerals, 0 to 100, reads them on request from any starting point.
5. Writes numerals from 1 to 100 in sequential order or on an ordered set of pictures for small blocks of numbers.
6. States, selects or writes the cardinal number of a structured group to 100.
7. Identifies what number comes after a given number, between two numbers, or before any given number for numbers to 100, with or without structured groups.
8. Selects which of two (or three) numbers is greater (greatest), smaller (smallest), for numbers to 100.
9. Places $>$ or $<$ between two numbers to indicate the greater or lesser with or without structured groups; to 100.
10. Places an X on the object with the specified ordinal position to "tenth."

Level C:

1. Reads and writes short sequences of numbers from any starting point to 200.
2. Reads or writes short sequences of numbers from any starting point to 200.
3. Supplies the number which is one more, one less, or in-between two given numbers, limit of 200.
4. Completes exercises for counting by 10's from any starting point. Limit of 200.
5. Completes exercises for counting by 5's from 0 to 200 starting at multiples of 5.
6. Completes exercises for counting by 2's from any starting point to 200. Identifies number as odd or even.
7. Completes mixed exercises for counting by 10's, 5's, or 2's. Limit of 200; starting only with multiples of the index number.

* I.P.I.: Individually Prescribed Instruction.

Level D:

1. Reads and writes numbers to 1000. Reads and writes short sequences backward or forward.
2. Skip counts by 3's to 1000 backward or forward.
3. Skip counts by 4's to 1000 backward or forward.
4. Converts decimals to fractions & words. Vice versa. Fills in number line. Tenths.
5. Converts decimals to fractions & words. Vice versa. Hundredths.

Level E:

1. Counts, reads, writes to 1,000,000, any starting point.
2. Identifies odd-even numbers. States, uses rules for addition, subtraction, multiplication 2 numbers.
3. Rounds numbers to 10's, 100's, for comparison and estimating answers in sample word problems.
4. Gives numeral for 2,3,4 place number written in words, writes 2,3,4 place number in words.
5. Writes decimal fractions for common or mixed fractions of 10 or 100 denominator -- vice versa.
6. Number words for mixed decimals to 1000ths. Vice versa.
7. Converts decimal fractions (to thousandths to other forms).
8. Orders mixed & pure decimals. To 100.001.

Level F:

1. Rounds numbers to nearest thousands, ten thousands, millions, for estimating answers.
2. Writes numerals for a 5,6, or more place number, writes words.
3. Locates prime numbers to 100 on a chart.

Level G:

1. Tests any number to determine if it is prime or composite.
2. Finds prime factors of given #.
3. Identifies numbers possible in base 5. Multiple choice questions about characteristics.
4. Writes base 5 numbers in expanded base 5 and 10.
5. Converts base 10 number to base 5. Vice versa.
6. Locates, writes negative numbers on number line, thermometer.
7. Illustrates use of negative numbers.
8. Writes numbers in scientific or other exponential notation. Positive powers.

Level H:

1. Identifies numbers in base 3 & 8. Changes numbers to base 10 and vice versa.
2. Adds and subtracts 1 & 2 digit numbers in base 2, 3, & 8 using expanded notation.
3. Adds and subtracts 1 & 2 digit numbers in base 2, 3, & 8 without expanded notation.
4. Identifies and uses the commutative principle for adding one and two digit numbers in base 5.
5. Identifies and uses the associative principle for adding more than 2 numbers (1 and 2 digits) in base 5.
6. Solves 1-step word problems which require adding and subtracting 1 and 2 digit base 5 numbers.

I.P.I. MATHEMATICS CONTINUUM:

Level I:

1. Identifies which numbers can appear in a base three, six, or seven system; writes numbers in the base in expanded notation; changes numbers written in base ten notation to numbers in this system and vice versa.
2. Adds and subtracts with one and two digit numbers using expanded notation in base three, six or seven.
3. Adds and subtracts one and two digit numbers without using expanded notation in bases three, six, or seven.
4. Identifies and uses the commutative principle for adding one and two digit numbers in base three, six, or seven.
5. Identifies and uses the associative principle for adding one and two digits in base three, six, or seven.
6. Solves one-step word problems which require adding and subtracting one and two digits in base three, six, or seven.
7. Uses repeated addition to solve multiplication problems in base three, five, six, or seven.
8. Solves multiplication problems with a number line for numbers in base three, five, six, or seven.
9. Completes a multiplication matrix for basic facts in base three, five, six, or seven.
10. Does multiplication of a one digit factor times a one or two digit factor for base three, five, six, or seven (may refer to above multiplication matrix).
11. Applies the distributive principle for multiplication in base three, five, six, or seven.
12. Solves one-step word problems requiring multiplication in base three, five, six, or seven.
13. Uses repeated subtraction to solve division problems in base three, five, six, or seven.
14. Solves division problems with a number line for numbers in base three, five, six, or seven.
15. Does division with a two or more digit product divided by a one digit factor for three, five, six, or seven base number (may refer to multiplication matrix).
16. Applies the distributive principle for division in working with numbers in base three, five, six, or seven.
17. Solves one-step word problems requiring division in base three, five, six, or seven.
18. (H1) Develops the binary counting system (base two) and performs addition, subtraction, multiplication, and division in base two.
19. (H2) Develops the counting system for base twelve or other base created by the student. Adds, subtracts, multiplies, and divides in this base.

ADDITION

Level A:

1. Associates objects in a 1 to 1 relationship. Equivalent-non equivalent groups.
2. Manipulates objects to illustrate add. subt. facts through 6.
3. Circles number to identify how many objects are in each of two sets and in both sets together.

Level B:

1. Writes the number of objects in each of 2 sets and the number of objects when put together. Sums of 12.
2. Circles or fills in the correct numerals for pictured addition statements. Circles or fills in numerals for pictured subtraction statements. Sums to 12.
3. Uses the word when given the symbol and the symbol when given the word for +, -, =. Circles numerals which indicate sum or addends in a number sentence.
4. Fills in numbers (missing sums and/or addends) to make "true number sentences" for pictured addition and subtraction situations.
5. Finds the sums and differences for addition and subtraction statements with "+" and "-" signs.
6. Selects "other names for numbers" by matching addition or subtraction expressions (N + M) or(N - M) with pictured groups or numberals to 12.
7. Writes = or $\neq$ sign to identify true or not true addition or subtraction statements. Creates true number sentences by correcting incorrect sentences, changing only one number. Sums to 12.
8. Given an addition equation, writes or completes a second equation to illustrate the commutative principle for addition. Sums to 12.
9. Completes addition and subtraction sentences with missing sums (differences) or addends associated with familiar number families. Sums to 12.
10. Solves or completes 1 step word problems with pictures. Sums to 12.

Level C:

1. Does column addition with two addends for any two or three digit numbers, no carrying. Checks addition problems by adding in reverse direction.
2. Solves column addition problems with three or more addends and sums to 20.
3. Places $>$, $<$, or = between two addition expressions to show their relationship. Sums to 18.
4. Adds three single digit numbers in two different ways to illustrate the associative principle for addition. Puts in parentheses to show which numbers are added first. Sums to 12.
5. Adds two numbers to sums of 20 using expanded notation.

Level D:

1. Mastery sums thru 20. Timed test.
2. Column addition 2 addends, 3 + digits. No carrying.
3. Finds missing addends. 3 single digits. Sums thru 20.
4. Uses words sum, addends -- labels part.

5. Adds,carrying to 10's using 2 digit numerals, 2 or more addends. To 200.
6. Adds, carrying to 10's/100's, using 3 digit numerals,, 2 or more addends. to 2000.
7. Adds, carry 10's, 100's using 3 digit numerals,, 2 or more addends. To 2000.
8. Finds sums, column addition. Using 3 or more addends of 1 digit. To 50.

Level E:

1. Column addition, no carrying, 3 or more digit numbers, more than 2 addends.
2. Uses commutative principle of addition.
3. Uses associative principle for addition to add 2 or more place numerals.
4. Adds with carrying for 4 or more place numerals with 2 addends.
5. Adds 2 mixed numbers to thousands (whole no.'s) and hundredths (decimals).
6. Solves multiple-step word problems.

Level F:

1. Adds -- carrying 4 or more place numbers, more than 2 addends.
2. Adds, 2 or more numbers with whole number parts and decimals to the millionths.

Level G:

1. Adds 2 negative numbers, uses number line or thermometer.
2. Adds negative and positive numbers. Uses number line or thermometer.
3. Adds any 2 numbers which are multiplied by the same base to the same positive power.

Level H:

1. Adds all combinations of negative and positive numbers (more than 1 digit) without using a number line.
2. Writes small whole numbers or decimal numbers in scientific notation using negative powers of bases 2 thru 10. Adds 2 numbers which are multiplied by the same base to the same negative power.
3. Adds numbers with decimal parts to the thousandths place or more.

Level I:

PLACE VALUE

Level A:

Level B:

1. Writes the numeral which names a structured group of up to 100 objects as ---tens and ---ones.
2. Selects a structured group to match a given number. Numbers to 99.
3. Writes the digit which is in the 10's or 1's place as requested for a given number. States the place value of a particular digit.

I.P.I. MATHEMATICS CONTINUUM:

Level C:

1. Writes the cardinal number for structured decimal groups to 200.
2. Places $>$ or $<$ between two numbers to show which is the greater or lesser. Numbers to 200.
3. Writes a given number as ---hundredths, ---tens, and ---ones and in expanded notation using the + sign. Numbers to 200.
4. Writes the digit which is in the unit's ten's, or hundred's place as requested for numbers to 200. States the place value of a particular digit.
5. Writes numbers to 200 in columns for hundreds, tens, and units according to the place value of each digit.

Level D:

1. Identifies place value of 1's, 10's, 100's, 1000's in words or numbers.
2. Uses $>$, $<$ to 1000.
3. Writes number before or after a given number or between 2 numbers to 1000.
4. Writes numbers in expanded notation. To 1000.
5. Regroups, renames numbers for borrowing/carrying.
6. Add and subtract problems related by multiples of 10.
7. Writes decimals in expanded notation. Words, fractions, decimals.
8. Identifies place value of decimals, words, fractions, decimals. To hundredths.
9. Place value chart. Decimals. To hundredths.

Level E:

1. Identifies place value digits to 1,000,000.
2. Writes numbers to 1,000,000 in expanded notation, words/numbers "+" signs. Place value chart.
3. Uses $>$ or $<$ to 1,000,000
4. Uses multiples of 10 to generalize multiplication and division facts. Uses factors to 5 x 10.
5. Identifies place value of mixed decimals to 1000ths.
6. Writes decimal as whole number plus sum of decimal part to thousandths place.
7. Place value chart for mixed decimals to 1000.001.

Level F:

1. Place value chart for 4 + digit numbers.
2. Writes 10 as a power. Identifies the base and exponent or power of a term.
3. Writes number with 1 non-zero digit as a whole number $<$10 times a power of 10, i.e., 7×10^3.
4. Writes a number, 1 thru 9 multiplied by itself a number times in exponential form.
5. Reads and charts decimal numbers to millionths.

Level G:

1. Makes place value charts in base 5 and 10 to compare systems.
2. Makes decimal place value chart with positive exponents. Fractions instead of negative exponents. Limit 10^9.

Level H:

1. Makes place value chart in base 2, 3 and 8 for comparison with base 10.

Level I:

SUBTRACTION

Level A:

Combined with Addition -- see Addition Level A.

Level B:

See addition Nos. 1, 2, 3, 4, 5, 6, 7, 9, and 10 -- Level B.

Level C:

1. Subtract problems--sums to 18.
2. Subtract 2 digits--no borrowing.
3. Finds missing addend--2 single digits.
4. Uses $>$, $<$ or = between subtraction expressions. Sums to 18.

Level D:

1. Mastery subtraction facts, numbers to 20.
2. Subtraction no borrowing -- 3 or more digits.
3. Subtraction borrowing 10's place -- 2 digits.
4. Subtraction borrowing 10's, or 100's -- 3 digits.
5. Subtraction borrowing 10's and 100's -- 3 digits.

Level E:

1. Subtraction with borrowing, 4 or more place numbers.
2. Subtraction 2 numbers, whole no. parts to thousands, decimals to hundredths.
3. Solves multiple-step word problems.

Level F:

1. Subtracts 2 decimal numbers with whole number parts and decimals to the millionths.

Level G:

1. Subtracts -- number from + number. Vice versa. Uses number line/thermometer.
2. Subtracts two -- numbers. Number line/thermometer.
3. Does subtraction with numbers written in exponential form with the same base number (2 thru 10) to the same positive number.

Level H:

1. Does subtraction with numbers written in exponential form with the same base (2 thru 10) to the same negative power.

Level I:

COMBINATION OF PROCESSES

Level A:

Level B:

Level C:

1. Adds, subtracts mixed sets of problems. No borrowing. No carrying. Vertical or horizontal form. Sums to 99.
2. Sums & differences in money, measurement, time & geometry problems. No unit conversion. Sums to 18.
3. Solves one-step problems adding and subtracting money, time & measurement values to 18.
4. Fills in $>$, $<$, $\neq$ in addition, subtraction problems using money, time and measurement values to 18. No unit conversion.
5. Inserts + or - to complete an equation.
6. Fills in missing addend in 2-step equations combining add. and subt.

Level D:

1. Addition, subtraction. Vertical or horizontal. Money, time and measurement. No carrying/borrowing. To 2000.
2. Same with carrying and borrowing.
3. Multiplication and division. Any earlier skills. Through 5 x 10.
4. Solves 1 or 2-step word problems.
5. Supplies missing operational signs.

Level E:

1. Adds, subtracts with/without carrying to 1,000 in any direction -- money to $1.00, time units.
2. Solves, equations -- "N" as a variable.
3. Multiplies, divides, combinations thru 9 x 9, 81 $\div$ 9.
4. Supplies missing sign $>$, $<$, = or $\neq$ for combinations of =, -, x, or $\div$.
5. Finds averages for numbers. To 1000.
6. Selects principle describing equation & vice versa.
7. Solves 1 or 2 step word problems with fractions to 1/8, time, money, measurement units, numbers to 1,000.

Level F:

1. Adds, subtracts to 1,000,000 all directions.
2. Mixed exercises in multiplication and division.
3. Supplies missing $>$, $<$, = or $\neq$ in addition, subtraction, multiplication, division problems.
4. One-, two-step word problems using all processes, fractions, money, time, measurement units.

Level G:

1. Adds, subtracts positive and negative numbers.
2. Adds, subtracts, multiplies and divides decimals to 2 places.
3. All processes all fractions.
4. Solves 1- and 2-step word problems.
5. Multiplies to find % of whole numbers.

Level H:

1. Solves multiple step word problem.
2. Solves insurance problems -- straight life, endowment insurance, etc.
3. Solves problems involving tax rate in mills per dollar.
4. Solves banking problems -- checks, depositing, withdrawing money.
5. Solves stock and bond problems.

Level I:

FRACTIONS

Level A:

1. Divides objects or sets of objects.
2. Identifies 1/2 of an object or set of objects.
3. Uses correctly and responds to use of the terms "whole" and "one half" in reference to objects or sets of objects.

Level B:

1. Identifies "1/2" or an object or set of objects. Limit of 12 objects.
2. Divides an object or set of objects in "half".

Level C:

*Throughout this unit, fractions in word and numeral forms will be taught together and then used interchangeably.

1. Divides a whole object into halves, thirds, or fourths and identifies an object divided into halves, thirds, or fourths.
2. Identifies 1/2, 1/3 and 1/4 of a whole object. Circles fraction which shows what part of an object is shaded. States that the terms one-half, one-third, and one-fourth mean "one of _____ equal parts."
3. Divides a set of objects into 2, 3, or 4 equal parts when instructed to divide a set into halves, thirds, or fourths and identifies sets of objects divided into halves, thirds, or fourths.
4. Draws a circle around 1/2, 1/3, or 1/4 of a set of objects and selects the fraction which describes the circled part of a given set.

Level D:

1. Identifies objects using 1/6, 1/8, 2/3, 3/4.
2. Divides sets of objects into parts.
3. Adds any 2 fractions with same denominator.
4. Adds 2 fractions, same denominator -- 1/2's, 1/3's, 1/4's, 1/6's, 1/8's only. Sums equal 1, 2, 3.
5. Identifies an equivalent fraction for a given fraction, using pictures.

Level E:

1. Uses all common fractions in dividing objects and sets. Responds to names.
2. Finds fractional parts of whole numbers giving a whole answer number.
3. Uses "denominator" to identify fraction parts.
4. Changes fraction to an equivalent fraction, with a different denominator, without the aid of pictures. Reduces fraction to lowest terms as a special case of the above.
5. Places $>$, $<$, or = between 2 simple fractions to show relationship. Reduces fractions to lowest terms.
6. Adds 2 or more fractions, same denominator. Performs subtraction of fractions. Reduces to lowest terms.

Level F:

1. Identifies an improper fraction and changes improper fractions to mixed fractions in lowest terms.
2. Performs simple addition, subtraction and multiplication with fraction having unlike denominators using picture regions, number lines, etc.
3. Finds greatest common factor for a set of numbers and uses the greatest common factor to reduce fractions to lowest terms.
4. Finds LCM for a set of whole numbers and finds the LCM for a given set of fractions.
5. Uses the algorithm for addition and subtraction of fractions, find LCD.
6. Performs addition, subtraction of fractions, unlike denominators. Reduces to lowest terms. Uses commutative, associative and inverse properties in checking problems.
7. Performs column addition, 2 or more simple fractions, like and unlike denominators. Reduces to lowest terms. Performs column subtraction.
8. Adds, subtracts fractions and whole numbers with improper fractions and mixed fractions. Answers, lowest terms.
9. Uses $>$, $<$, = or $\neq$ to show relationship between pairs of fractions.
10. Rearranges groups of fractions into ordered set.
11. Uses $>$, $<$ and = to show relationship between 2-step equations using fractional expressions with +, - and x.
12. Writes decimal equivalent for simple fractions (1/2, 1/4, etc.). Changes decimal equivalents to fractions.
13. Performs more complex multiplication of fractions including improper and mixed fractions. Finds common divisor, lowest terms.
14. Solves one-step word problems.

Level G:

1. Writes decimal equivalent for any proper or improper fraction and changes decimal equivalents to fractions.
2. Uses multiplication algorithm for multiplying all fractions.
3. Divides simple fractions, improper fractions, mixed fractions by using reciprocals. Lowest terms.
4. Solves problems with fractions raised to whole number powers less than 5.
5. Solves multiple-step word problems.

Level H:

1. Finds the value of a whole number raised to a fractional power.

Level I:

MONEY

Level A:

Level B:

1. Recognizes penny, nickel, dime.
2. Matches coins with numerical value, word cent used.
3. Recognizes quarter.
4. Finds values, pennies and nickels, use ¢ sign. Sums to 12 ¢.

Level C:

1. Matches a quarter with its numerical value or with value in other coins.
2. Finds the value of pennies, nickels, dimes, and quarters. Find equivalent coin combinations. Limit 99¢.
3. Identifies coins using pennies, nickels, dimes, and/or quarters. Totals a collection of coins and indicates if they are enough to buy an article. Limit 99¢.
4. Uses decimal point and $ in writing money values, for $ .10, $ .25, $1.00 and $1.50 only.

Level D:

1. Identifies 1/2 dollar, dollar, finds value, uses dollar sign.
2. Adds, subtracts money value. Horizontal/vertical. 2 addends. sums to $1.00
3. Totals coins, bills, greater, less, equal.
4. Writes money values using signs.
5. Identifies change in coins.
6. Solves one-step word problems.

Level E:

1. Identifies change in coins with purchase amounts up to $10.00.
2. Adds-subtracts money values, using cent and decimal notation.
3. Totals purchases, amounts less than $10.00. Indicates change. Counts out change starting with the total value of the purchase.

Level F:

1. Multiplies and divides money values using $ and decimal point. 1-digit multiplier/divisor. Limit $20.00.
2. Solves multiple-step word problems involving multiplication and division of money values.

Level G:

Level H:

Level I:

TIME

Level A:

Level B:

1. Reads numerals to 12 on clock face, orally.
2. Writes numerals to 12 on clock face.
3. Given only hour hand pointing between 2 numerals identifies interval as "before" or "after".

Level C:

1. Places an arrow on a "clock" number line to identify a given number of marks after the arrow to the end.
2. Counts marks and/or places an arrow on clock number line which is bent into a circle to form clock face. Responds to word "minutes".

Level D:

1. Selects matching clock faces.
2. Matches clock face to printed time.
3. Selects printed time to match clock face.
4. Draws hour, minute hand, draws both to show printed time.
5. Writes down other way to state times.
6. Matches time statements and clock faces.
7. Supplies minute count.
8. Supplies hour statement.
9. Writes time from clock face.
10. Draws time on face from statement.

Level E:

1. Identifies calendar units, # days in week, # days in each month. Completes calendars. Word problems. Writes given date in words and numbers or in numbers.
2. Reads any time on clock face, shows any time using clock face. Writes and reads time using appropriate vocabulary and punctuation.
3. Uses "morning", "afternoon", "night" dividing day at noon and midnight writes time and "A.M." and "P.M."
4. Finds minutes elapsed between 2 minute hand readings. Limit 2 hours. Calculates passage of time.
5. Solves problems adding/subtracting hours/half hours on clock face.
6. Identifies second hand. Reads time on clock with second hand. Says there are 60 seconds in a minute.
7. Adds/subtracts time units. One step problems. No regrouping. Limit 2 1/2 hours.
8. Problems in reading bus, train, plane schedules.
9. Addition/subtraction 2-3 time units. 1-2 regroupings. Seconds through years.

Level F:

1. Adds, subtracts units of time extending beyond 12:00.
2. Identifies equivalent values; decade, fortnight, score, century, leap year days.
3. Reads time -- 24 hr. clock.

4. Identifies time zones, works problems requiring time changes.
5. Identifies the change which daylight savings time makes in solving time problems.

Level G:

1. Names very small/very large time unit, e.g., nano second, millenium.
2. Gives practical instances for above.
3. Solves word problems.

Level H:

Level I:

SYSTEMS OF MEASUREMENT

Level A:

Level B:

1. Demonstrates comparative concepts.
2. Recognizes use of ruler, yardstick.
3. Ruler divisions, inches, 3 rulers = 1 yard..
4. Identifies dozen, one-half objects(s).

Level C:

1. Measures objects to nearest inch.
2. Solves measurement problems involving 12 inches in a foot. Differentiates measurements stated in inches and feet. Limit 3 feet.
3. States and shows cups per pint, pints per quart and reverse.

Level D:

1. Problems -- 3 ft. = 1 yd., 36 in. = 1 yd.
2. Uses equivalent liquid measures.
3. Word problems -- equivalent measures.
4. Measures length of lines or objects (up to 36 inches) nearest 1/4 inch.
5. Measures lines, objects to nearest 1/4 inch.

Level E:

1. Solves problems requiring conversion of tons into pounds, pounds into ounces, equivalent measures of ounces-pounds, pounds-tons.
2. Adds, subtracts, multiplies, divides, denominant numbers, uses regrouping to combine same units.
3. Reads speedometers. d = st problems.
4. Problems using temperatures. Above and below zero. C and F. No conversion.
5. Uses equivalent measures - feet, rod, yard, mile. Solves problems using these conversions.
6. Uses a ruler to measure in centimeters.
7. Measures lines - nearest inch and centimeter. Makes comparisons.

Level F:

1. Performs conversions between two metric length measures, mm, to km.
2. Performs conversions between metric and English measures.
3. Uses a meter stick for measuring.

Level G:

1. Weighs in grams, kilograms. Makes gram-kilogram conversions.
2. Converts metric to English. Vice versa.

Level H:

Level I:

GEOMETRY

Level A:

Level B:

1. Identifies figures, circle, square, triangle, rectangle.
2. Reproduces circle, square, triangle, rectangle from memory.

Level C:

1. Recognizes, names solids: sphere, cylinder, cube,cone, rectangular.
2. Reads directions with names of simple geometric figures.

Level D:

1. Identifies, curves, lines, segments, corners.
2. Labels points in line. Names line segments by end-points.
3. Draws pictured representations of solids or selects correct pictured representation when name of solid is given. Names pictured representations of solids.

Level E:

1. Identifies parts of a line segment. Names a line for any 2 points in it.
2. Identifies a right angle and names angle by three points.
3. Given words equilateral triangle, right triangle, quadrilateral, draws or selects figure. Vice versa.
4. Identifies lines which "look parallel".
5. Uses compass - draws circle.
6. Identifies intersecting lines, locates point of intersection.
7. Names points in a line, dot used as a representation of a point.
8. Measures line segment to nearest 1/2 and 1/4 inch.
9. Identifies lines which are perpendicular.

Level F:

1. Finds perimeters for polygons by measuring.
2. Uses square inch model to find areas of simple plane figures.
3. Makes conversions among square units.
4. Uses a 1 cubic inch square as a model to find volumes of simple solids. Solves volume problems.
5. Identifies plane geometric figures: trapezoid, pentagon, hexagon, and other regular polygons.
6. Locates circle parts: center, radius, diameter, chord, arc, semicircle, tangent.
7. Identifies a "ray" as a line segment with 1 end point and extending indefinitely in the other direction.
8. Measures line segment to the nearest 1/8 and 1/16 of an inch.
9. Uses compass to bisect a line segment, construct a line perpendicular to a given line.
10. Identifies the vertex of a triangle or angle.

Level G:

1. Identifies plane geometric figures; parallelograms, rhombus, convex and concave irregular polygons.
2. Finds perimeter for: parallelograms, rhombi, regular, irregular polygons by measuring.
3. Measures angles using protractor, draws angles. Identifies acute and obtuse angles.
4. Identifies value of "Pi," π, and can demonstrate its derivation.
5. Finds circumference of circle using the formulas: $C=\pi D = 2\pi R$.
6. Finds area of circle using $A = \pi r^2$
7. Uses formulas to find perimeter of: Square, rectangle and triangle.

Level H:

1. Finds surface area of prisms, cylinders, cones, pyramids.
2. Uses formulas to find volumes of familiam solids - prisms, cylinders, pyramids, cones, spheres.
3. Identifies congruent line segments and angles.
4. Constructs a line segment or angle congruent to a given line segment or angle.
5. Constructs parallel lines cut by a transversal. Identifies transversal.
6. Constructs perpendicular. Identifies right angles.
7. Identifies angles formed by a transversal. Demonstrates vertical angles (pairs), corresponding are congruent.
8. Uses the Pythagorean theorem to find the length of an unknown side of a right triangle.
9. Tabulates properties of plane and solid geometric figures.

Level I:

SPECIAL TOPICS

Level A:

Level B:

Level C:

1. Writes Roman numerals 1-30 reverse.

Level D:

1. Writes Roman numerals 1-100 and vice versa.
2. Reads thermometer--records temperature using degree symbol.
3. Reads bar graph to locate information.

Level E:

1. Writes Roman numerals for numbers to 500 and vice versa.
2. Reads distances from simple maps.
3. Reads and makes graphs-charts.

Level F:

1. Writes ratios as fractions, finds missing terms in a proportion.
2. Converts fractions and decimals to percents and vice versa.
3. Completes patterns for "what's my rule?" games, solves function problems by finding the rule.
4. Solves conversion problems using acre, sq. yd., rod, sq. mile.
5. Reads, makes graphs and charts, including bar, line graphs with fractions and decimals.

Level G:

1. Locates points on a coordinate plane and graphs ordered pairs.
2. Uses Venn diagrams to picture union and intersection of sets.
3. Solves simple algebraic equations with one unknown,
4. Solves word problems.

Level H:

1. Identifies integers, rational, irrational numbers, identifies sets which exhibit the closure property.
2. Completes mathematical and geometric logical deductive statements, using "if....., then....."
3. Makes charts to compare United States currency with that of other Countries.

Level I:

Henry H. Walbesser

Although I view with great alarm the growing number of negative effects on mathematics education that are being generated by the application of behavioral objectives, I remain a behaviorist. This paper is written with two objectives in mind: (1) to expose the educationally damaging consequences of adopting behaviorism and (2) to attempt to reestablish the credibility of behavioral objectives as a significant educational strategy.

The idea that instructional purposes be described in terms of reliably observable learner behavior has the appeal of precision. Being able to recognize which students have learned, and which have not, is a longstanding concern in the art of teaching. Using data gathered by one or more of the five senses as the source of evidence of learning seems logical. Evidence in the sciences uses this data base--the five senses. Why not education? Such a position appeals to reason and holds out the promise of successes that are equivalent to those enjoyed by the sciences. But the promise has not been the reality. Art does not transform to science so easily.

WHY WRITE BEHAVIORAL OBJECTIVES

The reasons offered for writing instructional purposes in terms of behavioral objectives are well known to most mathematic educators. A few of the most prominent are recited here:

Informing the learner about the purposes for instruction aids his learning and recall.

Knowing the behavioral objectives facilitates the designing of programs of individualized instruction.

Learning needs of individual students can be diagnosed more accurately when the expected learnings are precisely stated.

The success or failure of educators can be made accountable to the public in terms of the behaviors acquired by students.

Each learner can proceed at his own rate of acquisition when a course is a set of behaviors to be acquired.

The lists of expectations for behavioral objectives are extensive. However, advancing a claim neither makes it true or false.

Some of the commonly proposed effects of behavioral objectives are supported by research data, others are refuted, and some find mixed support. It is not the intention of this paper to debate these data or the associated claims. An assumption of this discussion is that there is at least some validity to the proposed effects of behavioral objectives, but such an assumption does not resolve the issue of whether behavioral objectives should be used as part of the mathematics curriculum.

CRITICISMS WITHOUT MERIT

Some criticisms of behavioral objectives are ridiculous. Although the following arguments against behavioral objectives (and that is a most generous appraisal of what follows) are often raised, the criticisms are frequently more emotional than thoughtful. Some critics say that behavioral objectives cannot be stated for affective or complex psychomotor competencies, and hence the whole idea should be abandoned. How absurd! Behavioral objectives merely state those instructional outcomes for which attainment can be acknowledged by one or more of the five senses.

*The Arithmetic Teacher, October 1972, pp. 418, 436.

If, indeed, there are some educational objectives whose achievement cannot be seen, heard, tasted, touched, or smelled, then behavioral objectives are not appropriate. One would wonder for such an objective, if any exists, just how it is decided that someone has achieved it and someone else has not.

Other critics argue that it is not possible to construct behavioral objectives for all instructional purposes, and therefore no behavioral objectives should be written. What spurious logic!

Still others argue that the "really important" educational purposes cannot be observed in the behavior of students until years after the students' attendance in class. Since this makes evaluation of acquisition very difficult, no behavioral objectives should be drafted. One response to such a position is simply to suggest that assessment be made whenever the instructor deems it appropriate, even if it is a lifetime away. All in all, it really is difficult to accept the premise that the attainment of every important instructional objective is years off.

Criticisms such as those offered in the preceding paragraphs do not settle the argument about the worth of behavioral objectives. However, the opponents of behavioral objectives will need to mount more persuasive arguments than these, if educators are to be convinced of the ill effects of behaviorism.

SIMPLICITY AS A PROBLEM

A portion of the difficulty being experienced with behavioral objectives can be attributed to the simplicity, as well as the elegance, of the concept. Competency in the construction of behavioral objectives is easy to acquire. Almost anyone can learn to write them. Mastery can be attained in a remarkably short time, usually of the order of four or five days. About all that is required for the successful acquisition of this competency is the adoption of a definition of a behavioral objective, followed by practice in writing statements that use the adopted definition. For example, using the Constructing Behavioral Objectives (Walbesser 1970; Walbesser, Kurtz, Goss, and Robl 1970) instructional program, demonstration that the individual has acquired the technical competence means that he can write statements that staisfy the six components called for by the definition - who, what performance, what materials the learner is given, who initiates the learner's performance, what criteria must be met for a response to be acceptable, and what special restrictions are to be included.

Being able to apply a definition is a necessary condition, but it is obviously not sufficient to achieve the description of important educational objectives. It is possible to draft technically correct statements - that is, to draft statements that satisfy the basic components of an accepted definition - while describing instructional purposes that are trivial. Perhaps it is because the acquisition of the technical competence of writing objectives is so simple to attain that mathematics educators who write objectives in this behavioral format ignore the question of quality in the behaviors being named. For whatever reason, most behavioral objectives that are available for mathematics curricula name trivial behaviors.

The hysteria that the pseudo-objectivity of behavioral objectives generates is no more clearly illustrated than in the present frenzy about performance contracting. Faced with the almost overwhelming difficulties of teaching complex communication and quantitative competencies, as well as the shrinking availability of resources, many school boards and many school administrators look to performance contracting as a panacea. Educators rush into a contract without careful appraisal of the puissance of the objectives offered by the contractor and often settle for mediocrity. The difficulty is not with the concept of behavioral objectives, but rather with the incompleteness of the concept to the problem.

I wish to add my voice to those who are seeking - no, demanding - quality in the behaviors offered as objectives for the mathematics curriculum. One means of examining for quality is to assess the puissance of the behaviors.

THE ABSENT DIMENSION OF QUALITY

An observation of some significance about the claims advanced by the proponents of behavioral objectives is that most deal with efficiency. Questions of worthwhileness and value have not ordinarily been a part of the behaviorist's dialogue. Inquiries such as "Is the behavior valuable? Will the acquisition of the behavior have any important payoff for the individual? Are these behaviors critical to a mastery of the power of mathematics as a problem-solving tool?" seem to be totally absent from the menu that behaviorists offer in their deliberations about behavioral objectives. Those who have spoken to this issue of quality - and quality is what we are speaking of - have generally been allied with the antibehaviorists. I, for one, would like now to publicly applaud the efforts of those like James Raths (1971), Robert Davis (1967), and J. Myron Atkin (1968) who have spoken to this issue with eloquence. For the sake of education, I hope it is not too late.

A MEASURE OF PUISSANCE

The puissance of a behavioral objective is conceptualized as a linear continuum - the most powerful behaviors being those that enable the learner to independently generate knowledge, and the least powerful behaviors, those that are associated with rote responses from the learner. Most behavioral objectives that are currently being written for the school curriculum are of low puissance. For this reason, behavioral objectives are now a negative force in contemporary mathematics education.

The request for quality is by no means equivalent to actually possessing a means of assessing quality. One procedure for assessing the puissance dimension of quality is presented here. This procedure has been influenced by Robert Gagne's work, especially the volume The Conditions of Learning (1970) and James Raths's writings, particularly his position on the importance of the instructional activity (1971).

Puissance is measured by first classifying the given collection of behavioral objectives along two dimensions--performance classes and complexity categories. As early as 1962, the concept of clustering performances into equivalence classes was advanced in an article published by the Science Teacher under the title "Curriculum Evaluation: Observations on a Position" (Walbesser 1966). In that article, ten performance classes were operationally defined. They include the now well-known (or infamous, depending on your point of view) set: Identity, Name, Order, Construct, Demonstrate, State a rule, Apply a rule, Describe, Distinguish, and Interpret. These performance classes have subsequently appeared in numerous collections of behavioral objectives including those proposed by Science--A Process Approach (American Association for the Advancement of Science and Xerox Corporation 1970), ISCB (Burkman 1970), and PRIMES (Berger 1970). The rows of the quality space matrix (See fig. 1) are nine of these performance classes; Interpret is omitted.

The nine performance classes are scaled with respect to the duration of the performance: Identify, Name, and State a rule are given a scale value of 1; Order and Distinguish are given a scale value of 3.

The columns of the quality space matrix concern the complexity of the performance being called for by the objective. The complexity categories follow.

1. Verbal chains are stimulus-response sequences where the response is made verbally, and the response chain is a rote one. For example, having a four-or five-year-old recite the alphabet or count from one to ten.

2. Nonverbal chains are stimulus-response sequences where the response has no verbal components and the response chain is a rote one. For example, constructing congruent line sements with a compass and straight edge, or performing a calculation using a desk calculator.

Fig. 1. Quality space matrix for three chapters from a popular seventh-grade mathematics textbook.

Performance classes		Complexity categories: Verbal chains (1)	Nonverbal chains (2)	Concepts (3)	Discrimination among concepts (4)	Principles (5)	Discrimination among principles (6)	Independently generating knowledge (7)
Identity	(1)	1	1	1	1	4	3	
Name	(1)			1		1	2	1
State a rule	(1)	1		2		2		
Order	(2)			1	1			
Distinguish	(2)				1			
Construct	(3)				1		1	1
Describe	(3)					1		
Demonstrate	(3)				1	3	1	
Apply a rule	(3)					2	2	

3. Concept is labeling an attribute shared by a specified collection and being able to generalize to instances not included in the original collection; the label is usually a noun. For example, the concept of two-ness, the concept of a mathematical operation, the concept of length.

4. Discrimination among concepts is selecting from among acquired concepts those appropriate to the solution of a problem or to answer a request. For example, having acquired the concepts of line, line segment, half line, and ray, count the number of rays in a given collection of figures.

5. Principle is combining two or more concepts to make a formula, law, or rule; or combining two or more concepts to make a new classification category. For example, $A = \pi \cdot r^2$, which combines the concepts area, equals, π, multiplication, radius, and squared; or "A triangle is a simple closed curve made up of three noncollinear line segments," which includes the concepts simple, closed, curve, three, noncollinear, and line segment.

6. Discrimination among principles is selecting from among acquired principles those appropriate to the solution of a problem or to answer a request. For example, an arithmetic problem involving several different operations, deciding the next step in a proof, or identifying an error in a computer program.

7. Independently generating knowledge requires that the learner create a new concept or principle by combining or modifying known concepts or principles, but the creation must be done independent of outside direction. For example, identifying and naming a pattern in a given set of numerical or geometric data, constructing a proof, or making a new generalization.

The implied ordering of the complexity categories is intentional. A complexity scale is hypothesized, moving from a value of 1 for verbal chains to a value of 7 for independently generating knowledge.

One common goal of all modern mathematics curricula is to assist the student in becoming an independent learner. Modern mathematics curricula are also marked by their attempts to increase the number of activities in which the learner is an active participant. The puissance value given

by the quality space matrix provides an assessment of how well any collection of behavioral objectives meets these two goals of active participation and learner independence.

One persistent and significant criticism (in this author's view) of behavioral objectives is that they concern themselves with the simple, unimportant, educational purposes. The quality space matrix and the puissance-value procedure provide one means of assessing such claims for any given collection of behavioral objectives.

The puissance measure is simple to compute. Consider the sample quality space matrix given in figure 1. Behavioral objectives for three chapters of a popular seventh-grade mathematics text have been catalogued. The publisher's literature about this text claims that it engages the student in much individual activity and leads him to discover several important mathematical ideas. When the puissance measure is applied, do these claims hold up?

The highest possible puissance measure has a value of 21, the lowest 1. Applications of the measure thus far suggest that 14-21 is a high puissance measure; 6-14 is modest; and 1-6 is low. The totals for the data in figure 1 are:

1st Row (1.1) + (1.2) + (1.3) + (1.4) + (4.5) + (3.6) = 48
2nd Row (1.3) + (1.5) + (2.6) + (1.7) = 27
3rd Row (1.1) + (2.3) + (2.5) = 17
4th Row (2.3) + (2.4) = 14
5th Row (2.4) = 8
6th Row (3.4) + (3.6) + (3.7) = 51
7th Row (3.5) = 15
8th Row (3.4) + (3.3.5) + (3.6) = 75
9th Row (3.2.5) + (3.2.6) = 66
Total = 321

$$\text{Puissance measure} = \frac{\text{total matrix value}}{\text{total number of objectives}} = \frac{321}{37} = 8.7$$

Despite the claims of the book publisher, the puissance measure eveals only a modest value for this set of objectives. In other words, the claims for the material are overrated.

Research has uncovered misrepresentations on both ends of the puissance scale, but the overwhelming number of low puissance measures obtained for sets of behavioral objectives among mathematics curricula suggest the immediate need for a revision of objectives or the content selected. Preliminary findings also suggest support for those critics of behavioral objectives who agree that the significant objectives of mathematics instruction have not been stated as behaviors. This author is inclined to revise such a statement by saying "have not <u>yet</u> been stated."

SUMMARY

The thesis that is being advanced by this paper has five components:

Behavioral objectives as a particular statement form are easy to draft.

Most behavioral objectives that are available today concern simple behaviors.

Behavioral objectives of this simple variety are attained by instructional materials that border on mechanical training activities.

The behaviors acquired from such activities have low generalized ability to develop independent problem-solving activity.

The curriculum makers and the educational proactitioners have deceived themselves in believing that all of the instructional purposes of a given course have been described when any set of behavioral objectives have been drafted for the course.

Make no mistake. This is not to argue that behavioral objectives should be abandoned as an educational technique. Rather, the foregoing observations are offered to alert the potential user to the present difficulty with such systems. There is another reason for this paper, namely, to provide a rationale for making a puissance estimate on each and every set of behavioral objectives that might become the instructional goals of a school curriculum.

REFERENCES

American Association for the Advancement of Science and Xerox Corporation. Science--A Process Approach, Parts, A-G. New York: Xerox Corporation, 1970.

Atkin, J. Myron. "Behavioral Objectives in Curriculum Design: A Cautionary Note." Science Teacher 35 (1968) 27-30.

Berger, Emanuel, director. Pennsylvania Retrieval of Information for Mathematics Education System (PRIMES). Harrisburg, Pa., Bureau of Research, Pennsylvania Department of Education, 1970.

Burkman, Ernest, director. "Seventh Grade Materials, Behavioral Objectives." In Intermediate Science Curriculum Project (ISCS). Morristown, N. J.: Silver Burdett Co., 1970.

Davis, Robert B "The Range of Rhetorics, Scale, and Other Variables," Journal of Research and Development in Education 1 (1967): 51-74.

Gagne, Robert M. The Conditions of Learning. New York: Holt, Rinehart & Winston, 1970.

Raths, James (a). "Teaching Without Specific Objectives." Educational Leadership 28 (1971): 714-20.

_____(b). "Worthwhile Activities." In Studying Teaching. Edited by James Raths, John R. Pancella, and James S. VanNess. Englewood Cliffs, N. J , Prentice-Hall, 1971.

Walbesser, Henry H. (a). Constructing Behavioral Objectives. College Park, Md.: Maryland Book Exchange, 1970.

_____(b). "Science Curriculum Evaluation: Observations on a Position." Science Teacher 33 (1966): 34-39.

Walbesser, Henry H., Edwin B. Kurtz, Larry D. Goss, and Richard M. Robl. Constructing Instruction Based on Behavioral Objectives. Stillwater, Okla.: Department of Aerospace and and Mechanical Engineering, Oklahoma State University, 1970.

ARE BEHAVIORAL OBJECTIVES THE ANSWER?*

Eugene D. Nichols

At several meetings of mathematics teachers and of college students, I have expressed some concern about the use of behavioral objectives as a way of prescribing the entire mathematics curriculum--or for that matter a curriculum in any subject area. A number of individuals have asked that I make some of my remarks available in print. This article is a response to those requests.

The concept of behavioral objectives is an interesting one and should be studied in depth by all teachers. Those who are interested in the balanced view of behavioral objectives would benefit by reading an article by Jack E. Forbes in the December 1971 issue of the Mathematics Teacher and an article by J. Myron Atkin in the Science Teacher of May 1968. At the request of several individuals, I am presenting in this article a case against the use of behavioral objectives as a means of prescribing educational goals.

First, however, I should like to make it clear that I do not defend those who are fuzzy and sloppy about the desired outcomes of their teaching. Much improvement in this area is needed.

However, the answer does not lie in spelling out all educational goals in terms of behavioral objectives. I have chosen eleven objections to the use of behavioral objectives for this purpose.

1. The primary responsibility of the teacher is not to shape the behavior of the student directly; rather, the teacher's responsibility is to develop and build in the student a reservoir of all that is needed to let the student choose from all alternative behaviors the one that is best for the given occasion. This reservoir will ordinarily consist of the skills, attitudes, and abilities necessary to make value judgments. Piaget, Bruner, and others have said this in many different ways. They consider the goals of education to be mainly those that prepare an individual to face new situations and solve problems that they have not encountered before. In other words, education should produce individuals who are able to adjust to, and deal effectively with, unfamiliar situations. To ask teachers to shape their teaching in terms of behavioral objectives spelled out in advance--and only those that are easily measurable--is to suggest that education is synonymous with training.

2. The concept of behavioral objectives leads many individuals, particularly administrators, to equate the totality of the goals of any curriculum with a set of behavioral objectives. In my judgment, to be clear about the goals of one's teaching is not synonymous with writing behavioral objectives. Cumulative side effects of teaching may be more important than the direct, planned-for goals. Also, not all aims of instruction are short term. The long-range aims may be among the most important ones. As a teacher, I am very uneasy about the claim by some that behavioral objectives can account for all aims of education. Deep down inside I do not believe that the individual's visible behavior reflects my total desire for my students. I feel that what a student is and what he thinks are also of importance. Therefore, what he does is not the whole measure of his educational worth.

3. In spelling out the entire curriculum in terms of behavioral objectives, one makes an assumption that there is a clear-cut relationship between the teaching of something and the behavior that is caused by the knowledge of it. This is far from the truth. There are too many unforeseeable side benefits that may be incidental, yet of crucial importance--even of much greater importance than the spelled-out objectives. Life, even in the classroom, is not as simple as the concept of behavioral objectives makes it out to be.

* The Arithmetic Teacher, October 1972, pp. 419, 474.

4. Behavioral objectives do not prescribe how a particular concept should be taught, nor do they offer any assistance to the teacher in how to teach that concept. At the time many of us have proposed heavy revisions in the elementary school mathematics curriculum, we have introduced, among other things, some new concepts. For example, we felt it desirable for children to explore such patterns as the commutative and associative laws. The emphasis was on explore. In practice, however, we found that many teachers had children memorize the patterns as well as the names for them. Any design of a curriculum should give thorough and serious attention to the how of the presentation of concepts. Thus, in suggesting that children should recognize instances of different operations to discover for themselves that certain operations do have this property and others do not. This important aspect of teaching is outside the realm of behavioral objectives.

5. Behavioral objectives must be prescribed in the greatest detail and in advance. Neither teaching nor life is that way. The unexpected is part and parcel of both, of teaching in the classroom and of life. As a matter of fact, I believe that any teaching from which the unexpected has been sucessfully eliminated is worthless. It is not teaching; it is training. Obviously, not all objectives are known in advance--and never can be. Many objectives evolve as teaching-learning situations develop. Every teacher must be sensitive to the continuous responses of the learners, to the questions posed, and to the interactions of students with students. Students' responses, in turn, suggest the next step for the teacher to take. This is an essential ingredient of teaching and one that cannot be mapped out in advance.

6. The issue of behavioral objectives has forced teachers to debate what is behavior and what is not--whether or not such verbs as think, consider, learn, discover, recognize, and many others describe behavior. In my judgment, such debates are not worth the energy expended. Obviously, teachers do want their students to be able to think, to learn, to understand, and sometimes even to understand fully. What these verbs mean is best clarified not be stating outcomes in behavioral terms, but rather by presenting examples of activities that students should be able to perform successfully.

7. A good match between what you test and what you have taught is a necessary thing. Measurement experts call it validity, but a prescription, in advance, of exactly how many items out of how many should be done correctly in order not to have to go through the entire learning sequence all over again is questionable. It is oversimplified and punitive. I think the teachers know exactly what is needed in cases of unacceptable performance: it is the personal attention of the teacher. The reason it is not done is that it is not humanly possible for the teacher who has to teach 150 or more students during each day to give this personal attention to those who need it. I have seen many instances of cases where the resources spent on hiring teachers to write behavioral objectives would be much better allocated if some additional teachers were hired to give this badly needed personal attention.

8. The assumption underlying the demand for specifying all outcomes of education in terms of observable behaviors assumes that an individual is what he does. Experience shows that frequently what an individual does not do is more significant than what he does. Silence does not qualify as one of the behavioral objectives. And yet silence can be worth a thousand words and a thousand behaviors.

9. The concept of behavioral objectives entails the notion that only measurable objectives qualify. More dangerously, teachers have been led to believe, in many instances, that those objectives that can be clearly defined and that can be easily assessed are the only important objectives for a program. I feel that in my teaching I do many things for my students that I consider not only worthwhile but essential: I indulge in humor, praise, scolding, expression of elation at the students' responses, expression of sympathy, and a thousand others--all in the context and pursuit of mathematics learning. As a teacher, I consider these things important and insist on having them as a vital and necessary part of my teaching. I would challenge the test experts to find ways to measure the effects of these kinds of things. I suspect that teachers

would refuse to leave them out of their mathematics teaching, and they would refuse to wait until ways of measuring them are found.

..10.. The idea of spelling out in advance what the student should be able to do as a result of my teaching a particular lesson appealed to me at first. I gave it an honest try. The students found it neither useful nor helpful. And in the course of testing this approach, it became clear to me why they found it so. To understand what one is expected to be able to do after going through a learning experience, one must know something about it at the outset; but behavioral objectives describe outcomes that are new and thus incomprehensible to the student if looked at prior to the learning experience. The student is naturally anxious to get into the study of the topic at hand and then be presented with situations in which to test what he has learned.

11. I have been involved in activities that resulted in some curricular innovations in mathematics. As I examine the kinds of things one must do in introducing innovations into a curriculum, I cannot help feeling that the demand for the statement of behavioral objectives at the outset frustrates the innovator and puts severe limitations on the activities that may result in a better curriculum. I found that a more productive way to begin is to have broad aims and then to proceed to refine these as one moves along. The proponents of behavioral objectives seem to miss this point, and it appears that they do not understand that such a back-and-forth refinement of educational aims is a necessary part of every curriculum-making effort. One should not start with a plan for changing the individual's behavior. The beginning point should be to examine the essential features of the discipline and the unique contributions it can make to the total development of the individual. The teaching process should be viewed in a global way, never losing sight of the total development of the student.

This article has been written in the hope that it will stimulate discussion and will prompt a serious reexamination of the entire issue of behavioral objectives.

GOAL-ORIENTED INSTRUCTION IN MATH *

For instruction to be of most benefit to pupils and most satisfying to teachers, it must be characterized by goal-oriented experiences in the classroom; that is to say, both teacher and pupils must be fully aware of the purposes which the instructional activity is designed to serve. Much controversy in education exists because of the vast difference between what we say we believe and what we actually do in terms of instructional practices.

To noticeably reduce the gap between theory and practice is not an easy task. The first essential is a clear delineation of goals or purposes. Operational decisions --grouping patterns, selection of materials and content, and evaluation--should all be based upon the values we hold and the goals we seek. The making of these decisions can be simplified only if the purposes they are to meet are clear cut, not hazy and undefined.

To what source does one go for clarification of goals or purposes? Statements made by the Educational Policies Commission have long helped to perform this service. Few are the educators who are not familiar with statements of objectives relating to self-realization, human relationship, economic efficiency, and civic responsibility. While the Commission's 1938 statement centered around these ideas, it is particularly interesting to note the change in point of view as evidenced in the 1961 statement. The central role of the rational powers is stressed throughout. Such powers "..... involve the processes of recalling and imagining, classifying and generalizing, comparing and evaluating, analyzing and synthesizing, and deducing and inferring. These processes enable one to apply logic and the available evidence to his ideas, attitudes, and actions, and to pursue better whatever ideas he may have."[1]

Though it is a relatively simple matter to achieve consensus on broadly stated goals, when it comes to interpretation of these goals with the anticipation of making resultant operational decisions, difficulties arise. What is really meant by rational powers, self-realization, civic responsibility? When we view goals as a basis for action decisions, it becomes distressingly clear that ambiguity and vagueness must move over and make room for painstakingly conceived specifics.

In a recent publication, Glen Heathers identified three major categories of goals which may help clarify our thinking:[2] content goals, process goals, and personal-social goals. The content goals, as these relate to any subject matter area, have to do with terminology, classification, information, explanatory theory, and technological application of information and theory. American education has typically emphasized the teaching of facts, although terminology and classification have also been stressed. It is this type of information-centered education that prompted Mortimer Adler to define formal education as "a process by which material passes from the notebook of the teacher to the notebook of the pupil, without passing through the head of either." At any rate, it is perfectly clear that an educated person is not one who merely has accumulated information--a walking encyclopedia. Facts are vehicles for the development of concepts and generalizations and, hence, rational thought. There has been relatively little concern for "....learning the general structure of ideas that orders knowledge of the field and provides the basis for understanding that knowledge and for employing it to serve practical ends."[3]

* From: Mathematics: Strategies of Teaching,
Buffie, Welch, and Paige; Prentice-Hall, 1968.

Goal-Oriented Instruction in Math

The process goals relate to student ability to acquire, interpret, evaluate and communicate knowledge as well as to develop basic fundamental skills. Educational reformers stress the point that a rapidly changing society must place a premium on one's ability to adapt to novel and unpredictable situations. Although there may not be complete agreement over what constitutes the process goals, Mr. Heathers' suggestions include: tool skills, critical and creative thinking, methods of inquiry, self-instruction, and self-evaluation. At the present time only the tool skills (reading, writing, counting, computing, etc.) seem to play a dominant role in American education.

Much controversy centers upon the extent to which the schools should assume responsibility for personality development, social behavioral patterns, and value formation. While goal emphasis is leaning more and more toward the development of intellectual powers, school must always be concerned with the personal-social goals because of the effect personal-social characteristics have upon learning. The development of an adequate and realistic self-concept is most important. Each child should be helped to develop a realistic understanding of both his strengths and weaknesses and then, more importantly, use his strengths to overcome his weaknesses when possible. As human beings strive toward selfhood, teachers have a responsibility to help children become increasingly more competent in self-direction, self-discipline, and self-evaluation. The responsibility for achievement of these particular goals, however, must be a shared one among home, church, and school.

As we continue to grow in our ability to state goals in more meaningful terms,[4] we will find ourselves drawn into closer confrontation with specific issues relating to goal accomplishment, i.e., grouping practices. If we accept the fundamental premise that our over-all educational goal is, indeed, the maximum development of the rational, creative being, every succeeding decision must find its roots in that acceptance.

If instruction is to be goal-oriented, one might quite logically inquire what all this suggests for the teaching of mathematics. The development of intellectual growth in mathematics, for example, is far too broad a goal for meaningful translation into action. Goals must be formulated and stated in more specific terms.

Years ago, the primary purpose of mathematics instruction was confined to developing computational skill in the fundamental arithmetical processes. Such an objective certainly would be categorized as a process goal (emphasis on the development of tool skills). Today our interpretation of process goals extends far beyond this very limited scope. In the past, the goal of computational facility greatly influenced the way in which the subject was taught, and emphasis was upon drill and practice. Do not be misled; today such a goal is still important. However, we would do well to recognize that today we have machines that can do calculations better and faster than any person. Some hand computers actually sell for less than a dollar. Therefore, other objectives must also serve as a focal point of instruction.

In the past decade or two, emphasis has been placed on the why as well as the how of computation. Mathematical understanding has taken its rightful place of major importance alongside the mechanics of arithmetical drill and skill. When understanding is firmly established, then drill and practice activities are capitalized upon to develop automatic mastery. Today's paramount concern for understanding has led to the identification of mathematical structures: those properties, principles, and laws which permeate all of mathematics. Who, today, has not heard of the commuta-

tive property, the associative property, the distributive property, or the identity elements? These ideas bind together the various elements of mathematics which are dealt with throughout the elementary school. A second major goal of instruction, then, is related to understanding mathematical content--the big ideas of mathematics as represented by the structure of the discipline. Another aspect of the content goals is an increased emphasis on related terminology. Much emphasis is placed upon learning and using proper mathematical vocabulary. As is true with all forms of language (and mathematics is a language), the more precise the vocabulary the greater is the possibility for effective communication and understanding. Once again, however, mere accumulation of information--understandings and terminology--could hardly be used alone to justify the teaching of mathematics (even if used in conjunction with the goal of computational efficiency). Content goals must be accompanied by still other goals.

As the case for a strong definition of content goals continues and strengthens, a parallel development is expected to evolve in evaluating and strengthening process goals. Emphasis in this area has traditionally centered on the fundamentals of computation. But today we are becoming equally concerned with helping children develop their rational powers and their ability to do critical and creative thinking. Teachers are capitalizing on the benefits of the inquiry, discovery, problem-solving approach to instruction, as well as techniques of self-instruction and self-evaluation.

Earlier, reference was made to "the development of rational power." Such a phrase has slightly different meanings depending upon the context in which it is found. In mathematics we are concerned with inquiry and creative thinking and problem-solving (all of which are process goals). Pupils are encouraged to approach problems or questions, each in his own unique way. A third major goal, therefore, is that divergent thinking be encouraged and its development deliberately sought. In the past, the usefulness of mathematics in practical matters was the most important factor in its vitality as a subject. Even more important today than the inculcation of this practical handling of the basic facts and fundamentals is the kind of thinking we wish to encourage and the kind of mind we hope to cultivate.

In order for pupils to become more creative in solving problems in unique ways, whether they be mathematical problems as illustrated below or mathematical problems in a social setting, they must develop an understanding of the structure of the discipline. It is this understanding and knowledge which makes possible unique and creative thought.

Picture this assignment: Solve the following problem and check it in as many different ways as possible:

$$n = 16 \times 25$$

Pupil solutions included:

25	16	25		16	
16	25	16		25	
150	80	150	(6 x 25)	80	(5 x 16)
25	32	250	(10 x 25)	320	(20 x 16)
400	400	400		400	

Goal-Oriented Instruction in Math

$n = 16 \times 25$
$= (4 \times 4)25$
$= 4(4 \times 25)$
$= 4(100)$
$= 400$

$n = 16 \times 25$
$= 16(5 \times 5)$
$= (16 \times 5)5$
$= 80 \times 5$
$= 400$

$n = 16 \times 25$
$= (10 + 6)25$
$= (10 \times 25)$
$+ (6 \times 25)$
$= 250 + 150$
$= 400$

$n = 16 \times 25$
$= 8 \times 50$
$= 4 \times 100$
$= 2 \times 200$
$= 400$

$n = 16 \times 25$
$= (16 \div 2)$
$\times (25 \times 2)$
$= 8 \times 50$
$= 400$

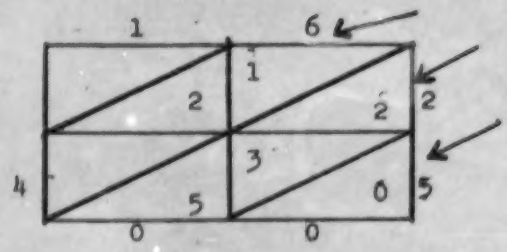
25
25
25
-
-
-
+
400

16
16
16
16
-
-
-
+
400

16
$25\overline{)400}$

25
$16\overline{)400}$

$n = 16 \times 25$
$= (2 \times 8)25$
$= 2(8 \times 25)$
$= 2 \times 200$
$= 400$



Close examination of the above will reveal pupil understanding of: (1) the commutative property of multiplication, (2) the associative property of multiplication, (3) the distributive property of multiplication, (4) the identity element, (5) inverse operations, (6) related operations, and (7) compensation.

Pupil understanding of the fact that numbers may be named in many different ways, i.e., 16 renamed as (4×4) and $(10 + 6)$, is also apparent. Such assignments give pupils an opportunity to provide evidence of their understanding and creative thought as well as to develop or further substantiate that mathematics is a man-made system and that man is in control of it and can use the system in whatever way he wishes to achieve his ends as long as he follows the rules of the game. To be creative, one becomes involved in thinking and doing in ways that are new or unique to him. Freedom to explore encourages creativity in thought and action and replaces the rigidity and conformity which all too often are characteristic of mathematics instruction.

Note then that the goal of creative thought is not something left merely to chance or simply a matter of providing opportunity for it to take place. We must channel our instructional efforts to this end if we wish attainment of this goal. Harry Emerson Fosdick said it best:

> No horse gets anywhere until he is harnessed. No steam or gas ever drives anything until it is confined. No Niagara is ever turned into light and power until it is focused, dedicated, and disciplined.[5]

It is important for all of us to realize that a fundamental understanding of the structure of mathematics is not an end in itself; rather, it is the relationship of this understanding to creative and divergent thought that provides in real value and meaning as an objective.

Goal-Oriented Instruction in Math

That the process goals, particularly those relating to inquiry and creativity, are being valued more and more today is an encouraging, not to be denied fact. That they may be considered even more important than the content-goals was implied years ago when Albert Einstein dared suggest that imagination was more important than information. The process goals then relate to the matter of helping children learn how to learn. Such goals further the development of such traits as originality, boldness, and flexibility. In the past, many of our schools, consciously or unconsciously, seemed to value conformity, timidity and rigidity. The focus was all too often on the "right answer" with little concern for the manner in which the answer was obtained.

The outstanding mathematics program of the present and the future will be one which has its strengths in creative and analytical abilities and fundamental understandings rather than mere assimilation and computation of indiscriminate facts and figures. Jerome Bruner, in his Notes on a Theory of Instruction, puts it well:[6]

> A body of knowledge....is the result of much intellectual activity. To instruct someone in those disciplines is not a matter of getting him to commit results to mind. Rather, it is to teach him to participate in the process that makes possible the establishment of knowledge. We teach a subject not to produce little living libraries on that subject, but rather to get a student to think mathematically for himself, to consider matters as a historian does, to take part in the process of knowledge getting. Knowledge is a process, not a product.

A fourth major goal recognizes the importance of attitude. Wherever goal-oriented instruction centers upon creativity and inquiry, children will also gain much in terms of personal enjoyment of learning. We should be as much concerned with helping children develop and maintain a zeal for learning as we are for them to acquire a specific quantity of learning. Not only is the knowledge we possess important, but also how we feel about the knowledge which has been acquired. Much of learning is an emotional experience. Knowledge itself can be used for good or evil, or perhaps not used at all. Attitude, as much as anything else, determines the direction that any one individual will go. We oftentimes wonder how many people actually realize how very close the relationship is between the formulating of sound positive student attitudes and the learning experiences which children encounter.

Changing goals affect the nature of instruction. As goals change so also must methods of instruction; otherwise, great inconsistency and ultimately conflict will be evident among purposes, instruction, and eventually, evaluation. Too many people view the modern mathematics program as little more than a change in content--emphasis on the big ideas (structure), greater emphasis on geometry, the introduction of other numeration systems and different number bases, or simply a matter of pushing content into lower grade levels. They do not recognize that it is the very basis of instructional goals which has changed, or perhaps only partial recognition is made and accepted, with the result that the learning experiences are focused narrowly, and unfortunately, on computational skills and mathematical structure. The other imminent danger is that the people responsible for formulating learning experiences for children will fully recognize and even philosophically accept the changes in goal-emphasis, but for one reason or another fail to relate and reflect this knowledge in instructional practice.

Goal-Oriented Instruction in Math

In Summary:

As changes have been made in curriculum and in instruction, concern for specifying clear-cut purposes or objectives has not been typically prominent. Recent instructional innovations and curricular reorganization have reflected continuing concern for the techniques of teaching: use of inquiry, discovery, and problem-solving approaches, and the grade placement of content. Emphasis upon the mechanical aspects of instruction, such as homogeneous grouping and multiage grouping, has served to divert debate into procedural matters. And yet, one must recognize that the most crucial and most sharply defined differences among those who seek to influence instruction are to be found in their concept of the purposes and goals of the mathematics program in the elementary school. It will be as these differences are resolved and objectives are defined that we will see marked educational improvement, and it is to this end that we must put our efforts.

[1]The Educational Policies Commission, The Central Purpose of Education (Washington, D.C.: National Education Association of the United States, 1961), p.9.

[2]Judson T. Shaplin and Henry F. Olds, editors, Team Teaching (New York: Harper and Row, 1964), pp. 349-351.

[3]Ibid., p.349.

[4]See Robert F. Mager's Preparing Instructional Objectives. Mager stresses the importance of stating objectives in behavioral or performance terms, such objectives being both observable and measurable. See also "Writing Instructional Objectives" by Thorwald Esbensen in the Jan. 1967 issue of Phi Delta Kappan, pp. 246-247.

[5]Jacob M. Braude, The Speakers Desk Book of Quips, Quotes, and Anecdotes (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963), p. 104.

[6]Jerome S. Bruner, "Notes on a Theory of Instruction," Toward a Theory of Instruction (Cambridge, Mass.: The Belknap Press of Harvard University Press, 1966), p. 72. Reprinted by permission of the publisher.

J. Briggs, F. Mettler, R. Denholm

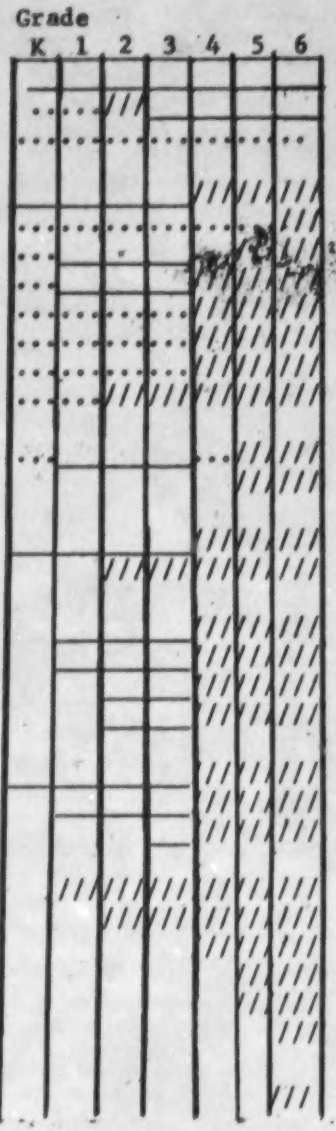
Coding:

Bars are printed according to the following code:

Interpretation:

_____	Appropriate content for grade level; found in basic text.
//////////	Appropriate content for grade level; found in supplementary text.
- - - - - - -	Content found in basic text; to be developed at teacher discretion (optional content).
xxxxxxxxxxx	Content found in supplementary text; to be developed at teacher discretion (optional content).
. . . . .	Content suggested for intuitive development.

TOPIC I: SETS (COLLECTIONS)

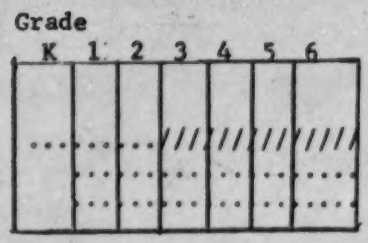


One-to-one correspondence and matching . . . . . .
One-to-two correspondence . . . . . .
One-to-many correspondence . . . . . .
Classification of sets:
  Equal (identical) and equivalent . . . . . .
  Nonidentical and nonequivalent . . . . . .
  Subset . . . . . .
  Empty. . . . . .
  Disjoint . . . . . .
  Not disjoint . . . . . .
  Finite . . . . . .
  Infinite . . . . . .
Operations on sets:
  Union . . . . . .
  Intersection . . . . . .
The number of a set (abstracting the idea of number from equivalent sets) . . . . . .
  n-notation . . . . . .
Operation on numbers deveoped from sets: . . . . .
  Addition explained from union of disjoint sets. . .
  Subtraction explained from set separation. . . . . .
  Equivalent sets to develop multiplication. . . . . .
  Division interpreted as set partitioning , ...
Properties of operation:
  Commutativity . . . . . .
  Associativity . . . . . .
  Distributivity . . . . . .
Set notation and symbolism:
  Braces and capital letter to name sets . . . . . .
  Commas to separate members of sets . . . . . .
  Empty set: $\emptyset$ or $\{\}$ . . . . . .
  Subset with Venn diagrams and subset symbolism ($\subset$).
  Union ($\cup$) and intersection ($\cap$) . . . . . .
  Notation of infinite sets. . . . . .
  Notation of finite set having many members $\{0, 1, 2, \ldots 100\}$ . . . . . .

(Cont'd. on next page)

*Adapted from: J. Briggs, F. Mettler, and R. Denholm, Implementing Mathematics Programs in California, A Guide K-8. Menlo Park: Pacific Coast Publisher, n.d., pp. 17-38. Reprinted by permission.

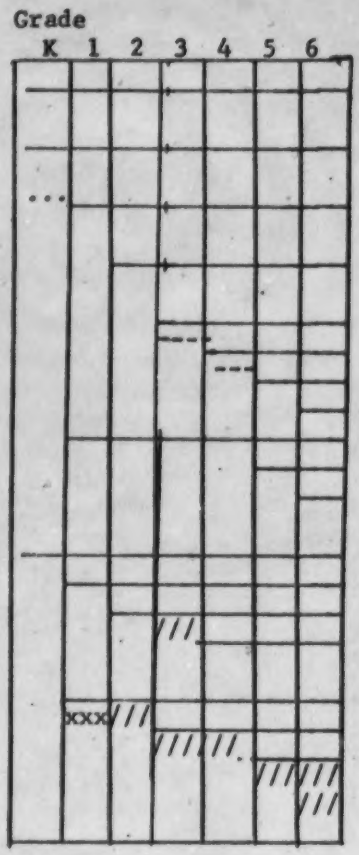
TOPIC I: (Cont'd)



Applications of sets:
- Geometry: Lines, etc., interpreted as sets of points.....
- Sets of ordered pairs.....

Solution sets.....

TOPIC II: NUMBER



Abstracting the idea of cardinal number from equivalent sets.....

Understanding sets of numbers:
- Counting: one through ten.....
- Whole (positive integers and zero)
  - Zero through one hundred.....
  - One hundred one through nine hundred ninety-nine.....
  - One thousand through niety-nine thousand, nine hundred ninety-nine.....
  - One hundred thousand through millions.....
  - Ten millions through hundred millions.....
  - One billion through hundred billions.....
- Fractional numbers (positive rational).....
- Decimals.....
- Negative integers.....

Ordinal numbers (position in a series):.....
- First through third.....
- First through tenth.....
- Eleventh through thirty-first.....
- Beyond thirty-first.....

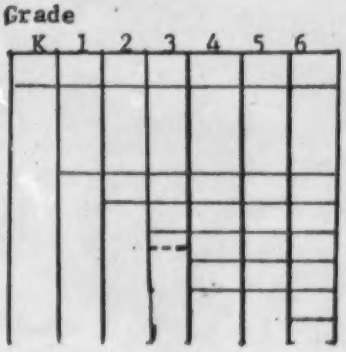
Number sequences:
- Counting by tens, twos, fives,.....
- Odd and even numbers.....

Rounded numbers.....

Prime numbers and factors.....

Composite numbers .....

TOPIC III: NUMERATION SYSTEMS



Number-numeral distinction; a number has many names .....

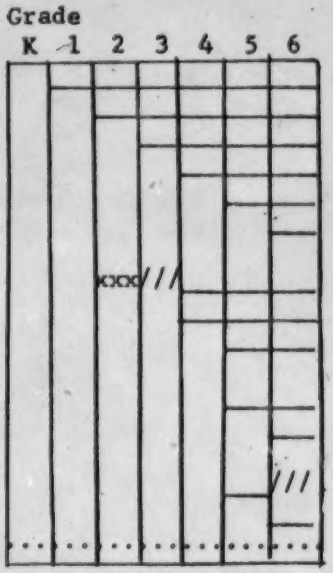
Place value: Reading and writing decimal numerals:.....
- 0 - 100.....
- 101 - 999.....
- 1000 - 99,999.....
- 100,000 - 9,999,999.....
- 10,000,000 - 999,999,999.....
- 1,000,000 000 - 999,999 999 999.....

(Cont'd on next page)

Interpretation:

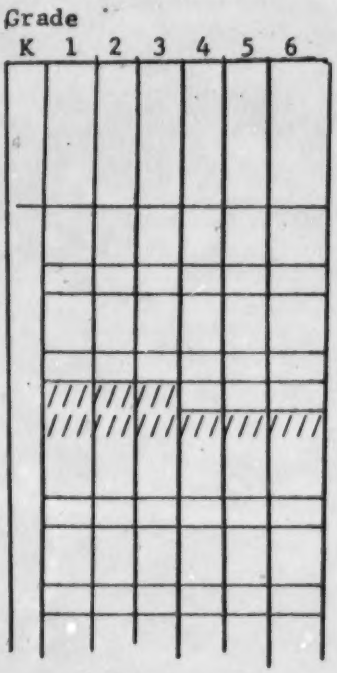
_____	Appropriate content for grade level; found in basic text.
//////////	Appropriate content for grade level; found in supplementary text.
----------	Content found in basic text; to be developed at teacher discretion (optional content).
xxxxxxxxxx	Content found in supplementary text; to be developed at teacher discretion (optional content).
....	Content suggested for intuitive development.

TOPIC III. (Cont'd)



- Expanded notation of decimal numerals:
  - Two-place numerals
  - Three-place numerals
  - Four-place numerals
  - Five-place numerals
  - Six-and seven-place numerals
- Exponential notation
- Roman numerals:
  - I, V, X
  - L, C
  - D, M
- Reading and writing decimal numerals:
  - Through .999
  - Through .99999
- Place value system in other bases:
  - Base five
  - Base six
- Historical development of numeration

TOPIC IV: ADDITION AND SUBTRACTION



(Note: In the basic texts for grade 1-3, "regrouping" is referred to as "carrying" in addition and "borrowing" in subtraction.)

- Sets to develop addition and subtraction
- Addition and subtraction combinations, sums through 18
- Mathematical sentences (horizontal notation)
- Addition and subtraction algorithms (vertical notation)
- Addition table (grid)
- Number line
- Cross Number Puzzle
- Properties of addition:
  - Commutative
  - Associative
  - "Zero" as the identity element for addition and for subtraction when "zero" is the subtrahend
  - Inverse relation of addition and subtraction

(Cont'd on next page)

TOPIC IV: (Cont'd)

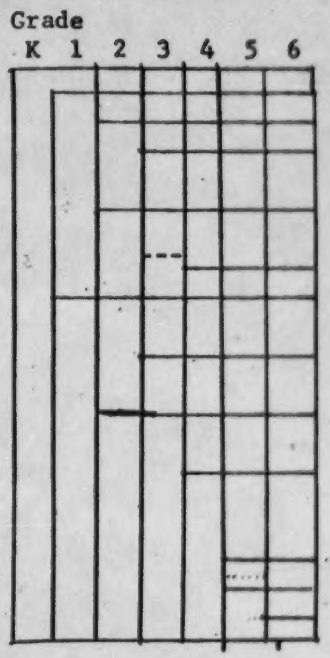
Addition and subtraction without regrouping
- (two-place).....
- Three-place.....
- Four-through seven-place.....

Addition and subtraction with regrouping
- Two- and three-place.....
- Four- through seven-place, regrouping in various places.....

Column addition.....

Addition with fractional numbers, sums not greater than "one".....

Addition and subtraction.....
- With denominate numbers.....
- Of fractional numbers, sums and minuends less than "two".....
- Of fractional numbers without limiting sums and minuends in horizontal and vertical form.....
- Of decimals through thousandths.....
- Of integers.....



TOPIC V: MULTIPLICATION AND DIVISION

Multiplication and division defined from set union and through set partitioning.....

Multiplication as repeated addition; division as repeated subtraction.....

Multiplication in rows and columns (arrays and Cartesian products).....

Multiplication and division combinations:
- Products through 25.....
- Products through 81.....
- In mathematical sentences (horizontal notation).....

Conventional multiplication algorithm (vertical notation).....

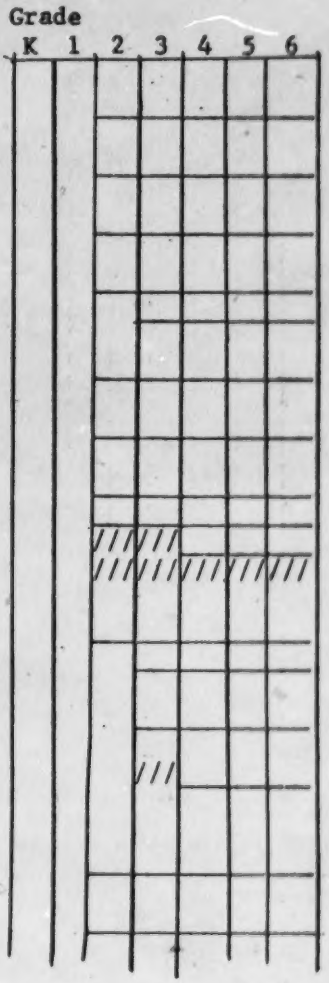
Multiplicative property of "zero" and impossibility of division by "zero".....

Multiplication table (grid).....

Number line.....

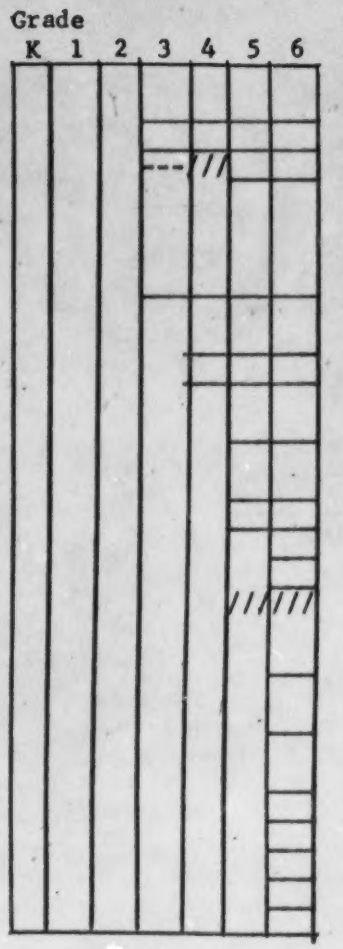
Cross Number Puzzle.....

Properties of multiplication:
- Commutative.....
- Associative.....
- Distributive property of multiplication over addition.....
- Distributive property of division over addition (only dividend may be renamed).....
- "One" as the identity element for multiplication and for division when "one" is the divisor.....
- Inverse relation of multiplication and division.....



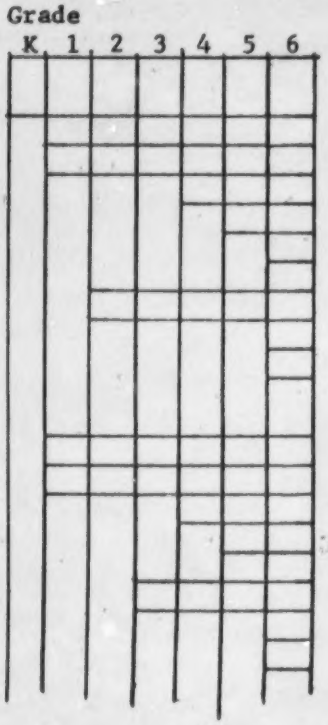
(Cont'd on next page)

TOPIC V: (Cont'd)



- Multiples of tens and hundreds as used in multiplication and division
- Vertical form and subtractive method of division
- Conventional division algorithm
- Multiplication with one-place multipliers and three-place multiplicands, division with one-place divisors and two- and three-place dividends
- Multiplication with two-place multipliers and three-place multiplicands
- Two-place divisors ending in 1, 2, 3, and 4
- Two-place divisors in subtraction form and in the conventional algorithm
- Quotient estimation by rounding divisors downward and upward
- Remainders expressed in fractional form
- Multiplication with three- and four-place factors
- Division with three-place divisors
- Prime factorization
- Multiplication and division with fractional numbers
- Removing common factors before multiplying (cancellation)
- Multiplication and division:
  - Of decimals
  - With denominate numbers
- Equivalent ratios
- Percents as ratios
- Exponential notation to express multiplication

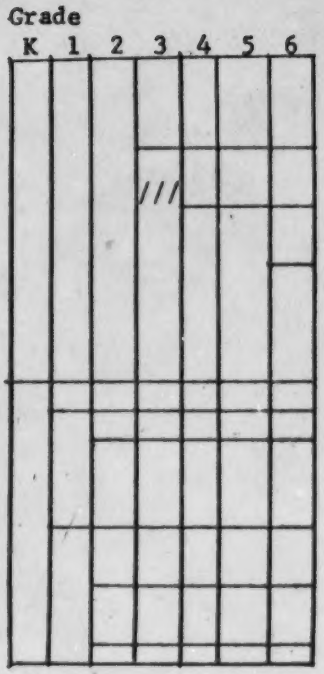
TOPIC VI: PROPERTIES OF THE OPERATIONS



- Commutativity:
  - Explained from union of sets
  - Of addition:
    - Of whole numbers
    - Of fractional numbers
    - Of decimals
    - Of integers
  - Of Multiplication:
    - Of whole numbers
    - Of fractional numbers
    - Of decimals
- Associativity:
  - Explained from union of sets
  - Of addition:
    - Of whole numbers
    - Of fractional numbers
    - Of decimals
  - Of multiplication:
    - Of whole numbers
    - Of fractional numbers
    - Of decimals

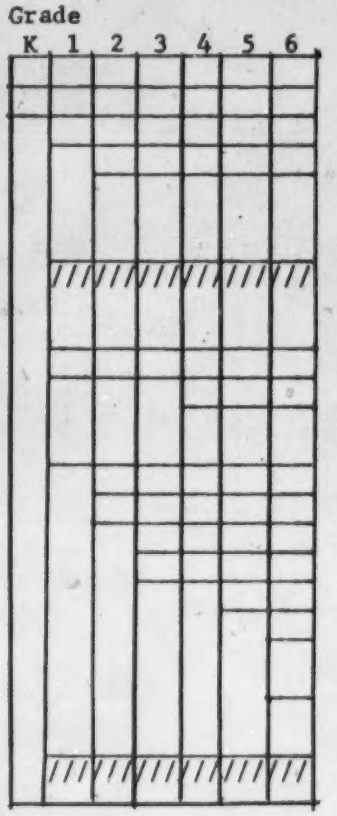
(cont'd on next page)

TOPIC VI: (Cont'd)



- Distributivity:
  - Of multiplication over addition (whole numbers).....
  - Of division over addition (whole numbers; only dividend may be renamed).....
  - Of multiplication over addition (fractional numbers and decimals).....
- Inverse relations:
  - Addition and subtraction as inverses explained from set union and set separation.....
  - Addition and subtraction as inverses.....
  - Multiplication and division as inverses.....
- Identity:
  - "Zero" for addition and for subtraction when "zero" is the subtrahend.....
  - "One" for multiplication and for division when "one" is the divisor.....
- Multiplicative property of "zero" and impossibility of division by "zero".....

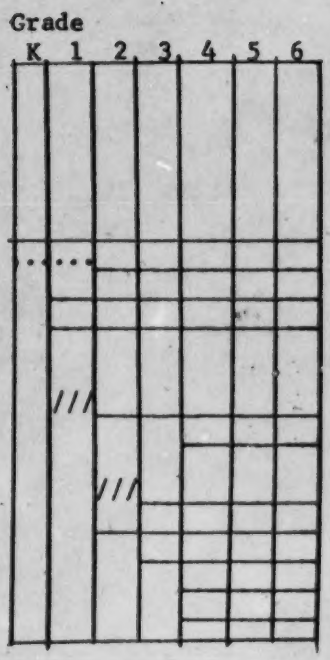
TOPIC VII: ORDER, RELATIONS, AND MATHEMATICAL SENTENCES



- Order in sets of numbers.....
- One-more, one-less concept.....
- Comparing whole numbers using sets and numerals.....
- Number sequences (odd, even,...).....
- Symbols:
  - Relationship
    - $=, >, <$ .....
    - $\neq$ .....
  - Grouping
    - Parentheses.....
  - Unknown number, operation, or relationship.....
  - Letters as variables.....
- Operations in mathematical sentences:
  - Adding and subtracting whole numbers.....
  - Multiplying whole numbers.....
  - Dividing whole numbers.....
  - Adding fractional numbers.....
  - Subtracting fractional numbers.....
  - Adding and subtracting decimals.....
  - Adding and subtracting integers.....
  - Multiplying and dividing fractional numbers and decimals.....
- Mathematical sentences involving:
  - Equalities, inequalities; open, closed.....
  - True, false.....

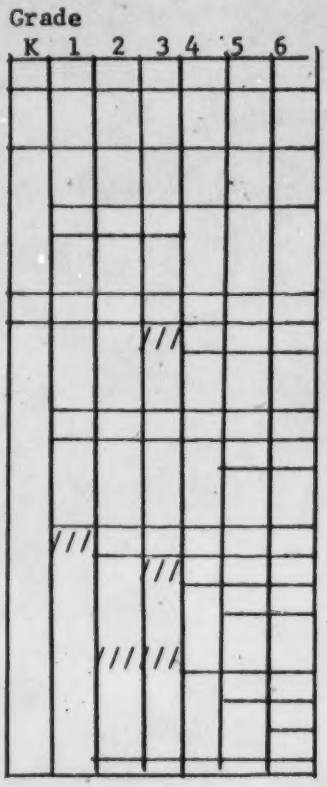
TOPIC VIII: PROBLEM SOLVING

(Note: It is recommended that the supplementary texts be used extensively to develop this topic in the primary grades.)



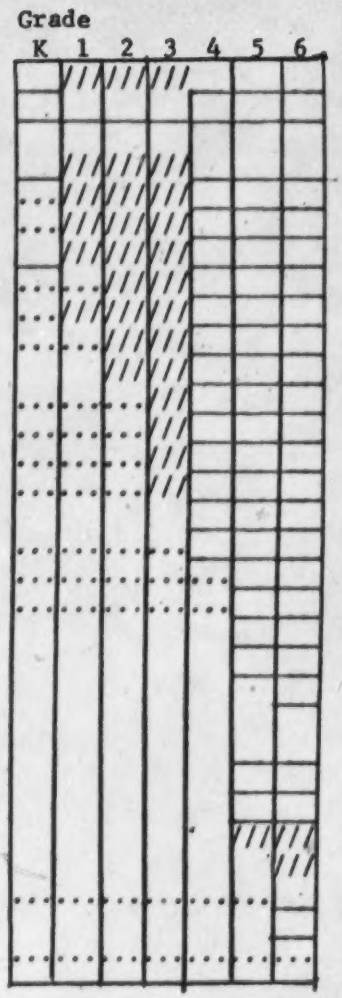
- Problem solving analysis procedures and techniques, including problem solving without numbers, many ways to solve a problem,.....
- Sketches and diagrams as problem solving aids.....
- Story problems.....
  - Read by pupils.....
- Composing story problems and translating into mathematical sentence form:
  - One-step problems.....
  - Two-step problems.....
- Story problems developed and written from mathematical sentence.....
- Evaluation of answers for reasonableness.....
- Estimation of answers in advance.....
- Story problems using data from graphs and tables.....
- Story problems involving inequalities.....

TOPIC IX: MEASUREMENT



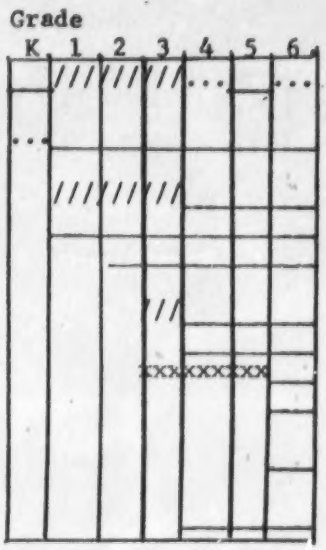
- Vocabulary of comparison.....
- Comparison of objects with each other and use of nonstandard untis.....
- Precision in measurement (approximate nature of measure and appropriate applications).....
- Symbol ≡ (is equal in measure to).....
- Measurement:
  - Money.....
  - Liquid.....
  - Weight.....
  - Time:
    - Clock.....
    - Calendar.....
    - Time zones.....
  - Length:
    - Foot, inch.....
    - Using ruler (inch, foot, yard,...).....
    - Map reading and scale drawing.....
    - Perimeter.....
  - Temperature:
    - Fahrenheit.....
  - Area.....
  - Angle.....
- Operations with denominate numbers.....

TOPIC X: GEOMETRY



- Geometric shapes in environment
- Geometric designs
- Two-dimensional shapes (circle, square, triangle, rectangle)
- Shape characteristics (measurement, inspection)
- Spatial relationships (distances)
- Comparing sizes, shapes, distances
- Construction of plane figures
- Interior and exterior regions
- Points, lines, points on a line
- Diameter concept
- Point in space
- Line, line segment
- Right angle ("square corner")
- Three-dimensional shapes (cube, sphere,...)
- Ray and angle
- Plane
- Intersection (lines, planes,...)
- Simple closed figures
- Area and perimeter
- Arc degrees
- Use of protractor
- Symbolism:
  - Line ($\overleftrightarrow{AB}$)
  - Line segment ($\overline{AB}$)
  - Ray ($\overrightarrow{AB}$)
  - Arc ($\overset{\frown}{AB}$), Triangle ($\triangle$)
  - Angle ($\angle$)
- Parallels and perpendiculars (lines, planes)
- Construction of three-dimensional figures
- Congruence (angles, triangles, line segments)

TOPIC XI: GRAPHING -- STATISTICS



- Tally marks
- Charts for reference, comparison, and record keeping
- Grids:
  - For record keeping by pupil
  - Addition table
  - Multiplication table
- Graphs:
  - Interpreting bar and picture graphs
  - Constructing bar and line graphs
  - Constructing pictographs
  - Superimposing one line graph on another
  - Comparing two sets of data and showing percent with circle graph
- Statistical concepts:
  - Average

MATHEMATICAL COMPETENCIES AND SKILLS ESSENTIAL FOR ENLIGHTENED CITIZENS*

E. L. Edwards, Jr.
Eugene D. Nichols
Glyn H. Sharpe

In March, 1970 the Instructional Affairs Committee proposed that an ad hoc committee write a set of objectives to define minimum competence in mathematics. Recognizing its responsibility to the profession as well as to the public to establish guidelines for the mathematical needs of all citizens, the NCTM Board of Directors accordingly appointed a committee to draw up a list of basic mathematical competencies, skills and attitudes essential for enlightened citizenship in contemporary society.[1] A report containing such guidelines was painstakingly prepared by the committee and is presented here. Even though the report does not represent NCTM official policy and may contain points with which individual members may disagree, it unquestionably contains much food for thought on a subject of great importance to society in general and to the mathematics-education community in particular. The entire report is therefore presented here for the consideration of the membership.

Following World War II, the crucial importance of mathematics in the technological and other aspects of life in the complex modern society has been unquestionably recognized. With it the question, "How much mathematics is a "must" for every citizen?" reappeared with new urgency.

The National Council of Teachers of Mathematics appointed the Commission on Post-War Plans and charged it with the task of answering this question. After completing its deliberations, the commission issued a checklist of twenty-nine (29) questions which constituted a basis for assessing one's basic competence in mathematics. "Guidance Report of the Commission on Post-War Plans" was published in 1947 in volume 40, number 7, of the MATHEMATICS TEACHER.

Since that time many changes have been introduced into mathematics. The very nature of the subject has changed. The introduction of new areas such as abstract algebra, functional analysis, and topology has resulted in what has been called a "revolution in mathematics."

Not until the 1950's, however, did these changes in the very nature of mathematics have an impact on secondary and elementary school curricula. Because of the revolutionary work of the University of Illinois Committee on School Mathematics, the School Mathematics Study Group, the Commission on Mathematics,

---

*Arithmetic Teacher, November, 1972. Pp. 601-607.

[1] The reader should recognize that this is not a list of minimum competencies, skills and attitudes to be attained by all students with an IQ of 70 or above; unfortunately, some of our students will not become enlightened citizens in contemporary society.

and other such groups, the curriculum underwent some drastic changes. New concepts appeared in textbooks and new approaches to teaching old concepts found their way into classrooms. Thousands of teachers returned to the universities and colleges to update themselves in the new concepts and methods. Writers of commercial textbooks, benefiting by the suggestions from experimental groups, introduced new concepts and approaches.

The realization in the late 1950's that the technology leadership of the United States had been challenged by others has given an additional impetus to the emphasis on the study of mathematics and the natural sciences. The demand for increased competence in mathematics has become a reality. New competencies and skills have been introduced into the required repertoire of the day-to-day activities of the individual.

The National Council of Teachers of Mathematics found itself with a challenge to define what might be considered the minimal mathematical competencies that are expected of the enlightened citizenship in contemporary society. Although it is impossible to find a universal agreement on such things, it is nevertheless the duty of this professional organization of mathematics teachers to propose what might be considered a reasonable approximation to such a statement of competencies. It is with this intent that this report is being offered.

Mathematics is one thing to some, another thing to others. Some enjoy it; others do not. Some seek to use it; others avoid it. Some individuals can get by with very little mathematical knowledge. In other words, the amount of mathematical skills that some individuals must have to "barely get by" in society may be very nearly zero. Present-day society collectively, however, needs a great deal of the most sophisticated kind of mathematics for its functioning. It is essential for its very survival. The highly complex problems of the technological society require complex mathematics to solve them. The demand, therefore, for increased competencies in mathematics has become a reality for many.

There are three basic ways to view mathematics:

1. Mathematics as a tool for effective citizenship and personal living.
2. Mathematics as a tool for the functioning of the technological world.
3. Mathematics as a system in its own right.

Each of these three aspects of mathematics is of significance. The first aspect is important for all. The second is essential to the scientist, the engineer---generally to those who make their living through professions that use mathematics as a tool. The third aspect is of concern to the professional mathematician. He sees in it a fascination and beauty that usually escapes the mathematically untrained mind.

In isolation, no one of the three aspects is of greater significance than another. In the total complex of society, each aspect is of significance.

Few educators will deny that past efforts in mathematics education have produced significant gains for many pupils. There are too many cases, however, where

many pupils leave our schools without the necessary skills to make them employable and to allow them to apply mathematics to help them solve the problems of daily life.

Some decisions are desperately needed to give guidance and direction to teachers and administrators concerning:

1. What mathematics every pupil must master to "barely get by" in contemporary society;

2. What mathematics is essential for full participation of an individual in contemporary society;

3. What teaching techniques and processes not only assure the acquisition of mathematical skills and competencies deemed essential but also convince pupils to be willing to use these skills once acquired.

The following listing of competencies and skills has been separated into three main categories.

In the first section, an attempt has been made to include examples of those skills and competencies that are felt to be necessary for the majority of adults in our society. This is not to imply, however, that every individual must master each concept listed. It is the hope of the NCTM that the reader will consult these guidelines and interpret them in light of his own situation. The listing should give some guidance to teachers of mathematics in establishing priorities. It is hoped that teachers will attempt to convince pupils by appropriate and reasonable objectives, activities and other techniques that the acquisition of essential mathematical skills and competencies is necessary for effective functioning in today's society.

In the second section, a few of the basic characteristics of the nature of mathematics as a system in its own right are presented. It is done with the hope that the individual curious about this aspect of mathematics as a discipline will find it helpful as he seeks to comprehend the basic nature of mathematics.

The third section should appeal to those individuals with the kind of inclinations discussed in the second section. These individuals are usually more inclined to seek a deeper understanding of the role of mathematics in society than those who are not mathematically inclined.

SKILLS AND COMPETENCIES

Individuals often find the need to use mathematics in everyday life and in many jobs that frequently require some technical application of mathematics. The following outline of content gives some indication, under each heading, as to what minimum "doing" skills are needed by the enlightened citizen.

1. Numbers and numerals

   a. Express a rational number using decimal notation.
   b. List the first ten multiples of 2 through 12.
   c. Use the whole numbers in problem solving.
   d. Recognize the digit, its place value, and the number represented through billions.
   e. Describe a given positive rational number using decimal, percent, or fractional notation.
   f. Convert to roman numerals from decimal numerals and conversely (e.g., date translation).
   g. Represent very large and very small numbers using scientific notation.

2. Operations and properties

   a. Write equivalent fractions for given fractions, such as 1/2, 2/3, and 3/5.
   b. Use the standard algorithms for the operations of arithmetic of positive rational numbers.
   c. Recognize and use properties of operations (grouping, order, etc.) and properties of certain numbers with respect to operations ($a \cdot 1 = a$; $a + 0 = a$; etc.).
   d. Solve addition, subtraction, multiplication and division problems involving fractions.
   e. Solve problems involving percent.
   f. Perform arithmetic operations with measures.
   g. Estimate results.
   h. Judge the reasonableness of answers to computational problems.

3. Mathematical sentences

   a. Construct a mathematical sentence from a given verbal problem.
   b. Solve simple linear equations such as:

      $a + 3 = 12$ $\qquad$ $16 - n = 4$
      $n/3 = 7$ $\qquad$ $4a - 2 = 18$

   c. Translate mathematical sentences into verbal problems.

4. Geometry

   a. Recognize horizontal lines, vertical lines, parallel lines, perpendicular lines and intersecting lines.
   b. Classify simple plane figures by recognizing their properties.
   c. Compute perimeters of polygons.
   d. Compute the areas of rectangles, triangles and circles.
   e. Be familiar with the concepts of similarity and congruence of triangles.

5. Measurement

   a. Apply measures of length, area, volume (dry or liquid), weight, time, money and temperature.

5. Measurement (continued).....

   b. Use units of length, area, mass and volume in making measurements.
   c. Use standard measuring devices to measure length, area, volume, time and temperature.
   d. Round off measurements to the nearest given unit of the measuring device (ruler, protractor, thermometer, etc.) used.
   e. Read maps and estimate distances between locations.

6. Relations and functions

   a. Interpret information from a graphical representation of a function.
   b. Apply the concepts of ratio and proportion to construct scale drawings and to determine percent and other relations.
   c. Write simple sentences showing the relations $=$, $<$, $>$, and $\neq$ for two given numbers.

7. Probability and statistics

   a. Determine mean, median, and mode for given numerical data.
   b. Analyze and solve simple probability problems such as tossing coins or drawing one red marble from a set containing one red and four white marbles.
   c. Estimate answers to computational problems.
   d. Recognize the techniques used in making predictions and estimates from samples.

8. Graphing

   a. Determine measures of real objects from scale drawings.
   b. Construct scale drawings of simple objects.
   c. Construct graphs indicating relationships of two variables from given sets of data.
   d. Interpret information from graphs and tables.

9. Mathematical reasoning

   a. Produce counterexamples to test the validity of statements.
   b. Detect and describe flaws and fallacies in advertising and propaganda where statistical data and inferences are employed.
   c. Gather and present data to support an inference or argument.

10. Business and consumer mathematics

   a. Maintain personal bank records.
   b. Plan a budget including record keeping of personal expenses.
   c. Apply simple interest formulas to installment buying.
   d. Estimate the real cost of an article.
   e. Compute taxes and investment returns.
   f. Use the necessary mathematical skills to appraise insurance and retirement benefits.

THE NATURE OF MATHEMATICS

An individual who is inclined toward the study of the structure of mathematics would do well to consider those attributes of mathematics that deal with this aspect of mathematics. Some of these attributes and the abilities associated with them are discussed below.

1. The deductive nature of mathematics. Here the concern is with the question of what follows by force of logic from known truths. The derivations of new conclusions take the form of "if.....then" statements. The central issue is whether the statement following "then" is a logical consequence of the statement, assumed to be true, following "if."

2. In relation to attribute one, an individual should be able to carry through a consistent argument. Proofs of theorems constitute a very important part of mathematics. It has been demonstrated that some very young children are capable of proving simple theorems. Those individuals who display this ability should be provided with opportunities to experience success in this kind of mathematical activity. For example, establishing in a logical way that the sum of two even numbers is an even number would be a very satisfying experience to a child with such inclinations.

3. An individual with an inclination toward this aspect of mathematics should be able to differentiate between a valid argument and an invalid one. Theorems are established by proceeding from assumptions to conclusions by means of a series of logical implications. It is important to be able to recognize arguments that have fallacies in them. One should appreciate the fact that liking a conclusion does not imply that it has been established validly.

4. An individual of this type should be familiar with the basic properties of operations on numbers. For example, such properties as commutative and associative of addition of real numbers should be a second nature to him. For those individuals, it is not sufficient to be skillful in the use of basic operations---they should be able to achieve economy in computations by making use of basic properties. The distributive property, for example, is very useful in simplifying computations.

5. These individuals should be able to verify whether or not a given system possesses given properties. They should study all the properties that a set of numbers under given operations should possess in order to constitute a particular system. For example, they should be able to verify whether a given set of objects is a field under two given operations.

6. Such individuals should be able to recognize that various concepts and operations are related to each other; for example, subtraction and addition are inverse operations. Having some idea of the ways in which concepts and operations are related to each other eliminates the necessity of memorizing many isolated facts. Experience and training in observing relations between entities aids retention and enhances the development of observational skills.

7. Such individuals should be able to perceive patterns displayed in sequences. Observations of patterns is a skill that finds applications in many areas of human endeavor. Mathematics is an ideal subject in which to develop this skill.

THE ROLE OF MATHEMATICS IN SOCIETY

The individual with the kinds of inclinations discussed in the previous section is also inclined to seek a deeper understanding of the role of mathematics in society than is usually the case with those who are not mathematically inclined. This implies the following:

1. Knowing the ways in which computers are used in sciences, technology, business, and government. Computers are indispensible for the functioning and the very survival of present-day society. The complexity of the present problems requires the use of computers. They are useful because of their speed and their ability to store a multitude of facts. The speed with which millions of facts can be processed is essential in complex decision-making situations. One must guard, however, against the belief that computers are capable of solving all problems, They should not be attributed the complex ability of independent thinking and decision making.

2. Recognizing the evolutionary development of mathematics by noting the historical milestones in the development of mathematical ideas. These mathematical ideas served man in solving his problems and aided him in the control of his environment. Beginning with the invention of numbers for the purposes of counting through the invention of irrational numbers for describing lengths man has conceptualized mathematical things which at first served exclusively his utilitarian purposes. Both the invention of a specific mathematical tool and its conceptualization into an abstract system are essential ingredients of the development of mathematics.

3. Being aware of the great frequency with which mathematical skills are used by individuals in their daily lives. To appreciate the importance of mathematics in daily life, one might pose a question as to what would happen if mathematics disappeared. If number of geometry disappeared, would man be able to go about completing successfully his daily chores?

4. Recognizing that there are problems that, by their nature, do not lend themselves to solution by mathematical methods. Individuals at times make decisions that are primarily based on feelings and emotions, and they should recognize that such bases for decisions, though they may be valid, are by their very nature not amenable to mathematical analysis.

5. Recognizing that some professions require knowledge of the most sophisticated and complex mathematical techniques. A young individual who, because of a demonstrated high aptitude for mathematics, may desire to train for a profession that has high requirements of mathematical competence should apprise himself of these requirements and acquire the skills necessary for the given profession. He should realize that many years of mathematical training may be necessary.

6. Being aware that mathematics finds direct applications not only in the natural sciences but in the behavioral and social sciences and arts as well. Many psychologists as well as individuals concerned with humanities make increasingly greater use of mathematical techniques in developing their disciplines. For example, computers are used to analyze and understand the factors that influence human behavior. Since there is a large number of variables involved in such situations, this analysis and understanding could not be achieved without the electronic computer. Environmental problems, because of their complexity, will demand more and more sophisticated mathematical models for their solution.

CONCLUSION

The basic mathematical competencies and skills essential for enlightened citizens are determined by the needs of society at a given time. In our rapidly changing technical society, many factors bear upon what are and what will be the required competencies and skills. Educators charged with maintaining a contemporary basic mathematics program must be aware of these factors and give them consideration in their efforts to design effective programs.

Ours is the age of technology. A lessening need for individual computational skills has resulted from the universal availability of calculators. Computers have largely obviated the need for paper and pencil computations, but they have increased the need for formulating careful and reasoned generalizations. As other devices become available, their influence on society must be reflected in basic mathematical competencies and skills identified as those needed by enlightened citizens.

The educational level of our society is constantly changing. A larger percent of the school-age population is enrolled in school; the number of years students remain in school is on the increase. The needs of citizens, therefore, will change as the educational level compounds itself.

Ingenuity in devising individual means of checking one's conclusions for "reasonableness" is a most valuable aid to effective living and is a skill that mathematical training can greatly sharpen. An enlightened citizen must logically and judiciously cope with the constantly increasing bombardment of statistics, facts, and figures. As all teachers and students become more aware of the demands for responsible, self-reliant judgments, the central role of mathematics in making such judgments becomes more apparent.

An enlightened citizen is qualified for employment. Employment opportunities are and will change both in nature and in requirements of personnel. Surely the degree to which an occupation is mathematically oriented will determine the extent of the basic mathematical skills and competencies required of individuals employed in that occupation. The aggregate degree of mathematical orientation of the job market, in turn, influences what is considered a basic level for all citizens.

Technology, the educational level of society, and occupational requirements are examples of factors that influence the basic mathematical competencies and skills

needed by enlightened citizens. Some other factors are probably manifested; still others will evolve from their influence. It is therefore critical that we continually examine the programs designed to improve basic mathematical competencies and skills in future years. It is also the hope of the National Council of Teachers of Mathematics that professional groups concerned with mathematical education for all citizens will constantly strive to interpret the factors influencing change, seeing these in relation to their implications for the mathematics curriculum.

The Goals of Mathematics *

Identifying the goals of mathematics may appear a major task at first. Fortunately, however, there has been considerable work done in this area already. Donovan A. Johnson and Gerald F. Rising have outlined goals for mathematics, and their conclusions appear below: [1]

> In selecting appropriate goals for mathematics instruction, we must take into account not only the needs of society, but also the mathematical needs of our students. Almost every committee or commission that has worked on revising the mathematics curriculum has stated certain basic mathematical needs. The following list summarizes these needs:
>
> 1. The student needs to know how mathematics contributes to his understanding of natural phenomena.
>
> 2. He needs to understand how he can use mathematical methods to investigate, interpret, and make decisions in human affairs.
>
> 3. He needs to prepare for a vocation in which he utilizes mathematics as a producer and consumer of products, services, and arts.
>
> 4. He needs to understand how mathematics, as a science and as an art, contributes to our cultural heritage.
>
> 5. He needs to learn to communicate mathematical ideas correctly and clearly to others. Communication is a tool basic to all civilizations.
>
> While it is not a specific mathematics objective, the teacher should not lose sight of the fact that the student needs to understand human relationships and to develop a personal value scale which respects the rights, needs and achievements of others.

By analyzing selected "reports, recommendations, and standard examinations issued in the United States in recent decades", the Committee of International Study of Achievement in Mathematics identified the following ten important mathematical behaviors: [2]

1. Ability to remember or recall definitions, notations, operations and concepts.
2. Ability to manipulate and compute rapidly and accurately.
3. Ability to interpret symbolic data.
4. Ability to put data into symbols.
5. Ability to follow proofs.
6. Ability to construct proofs.
7. Ability to apply concepts to mathematical problems.
8. Ability to apply concepts to non-mathematical problems.

* Excerpt form Sund & Picard: Behavioral Objectives and Evaluation Measures: Science and Mathematics, Merrill, 1972, pp. 15-18.

The Goals of Mathematics

9. Ability to analyze and determine the operations which may be applied.

10. Ability to invent mathematical generalizations.

The Cambridge Conference on School Mathematics has had a great impact on mathematics education because of the boldness and scope of its recommendations. In its original report, published in 1963, the committee listed these goals for mathematics: [3]

1. Lest there be any misunderstanding concerning our viewpoint, let it be stated that reasonable proficiency in arithmetic calculation and algebraic manipulation is essential to the study of mathematics. However, the means of imparting such skill need not rest on methodical drill. We believe that entirely adequate technical practice can be woven into the acquisition of new concepts. But our belief goes farther. It is not merely that adequate practice can be given along with more mathematics; we believe that this is the only truly effective way to impart technical skills.

2. Familiarity is our real objective. We hope to make each student in the early grades truly familiar with the structure of the real number system and the basic ideas of geometry, both synthetic and analytic.

3. Mathematics is a growing subject, and all students should be made aware of this fact. This recommendation is not made merely because we feel that every educated person should know the fact, but also because the knowledge that there are unsolved problems and that they are gradually being solved puts mathematics in a new light, strips away some of its mystique, and serves to undermine the authoritarianism which has long dominated elementary teaching in the area.

4. The building of confidence in one's own analytical power is another major goal of mathematics education.

5. The function of language is to communicate. In mathematics its function is to communicate with extraordinary precision; it is inevitable therefore that mathematics requires some special terminology. Special terms are good or bad exactly according to their effectiveness in communication, and the same applies to special notations and symbols. This principle must not only guide textbook writers, it must be brought home to the student.

6. To foster the proper attitude toward both pure and applied mathematics we recommend that each topic should be approached intuitively, indeed through as many different intuitive considerations as possible. In such a program, the student must be kept informed of where he stands. A curriculum which oscillates between logical rigor and guesswork can be confusing unless the student knows the level at all times. To present mathematics entirely in the rigorous deductive spirit not only precludes any possibility of applying mathematics, it is dishonest, even as a picture of contemporary pure mathematics. We hope that many problems can be found (we know a few) that read, "Here is a situation---think about it---what can you say?"

The Goals of Mathematics

7. Another goal of our program is the inculcation of an understanding of what mathematics is (and what it is not). We need not here belabor the point that the man in the street has considerable misinformation on this point; suffice it to say that this misunderstanding frequently seems to take the paradoxical form of ascribing both too much and too little power to mathematics.

8. While everyone should know about the wide range of topics suitable for mathematical analysis, it is almost equally important to understand the limitations of mathematics. Every application of mathematics depends on a model, and the value of a deduction is more an attribute of the model than it is of mathematics. We believe that students can be made aware of the distinction between the real world and its various mathematical models; in this we can look forward to cooperation from the sciences.

In a later conference, this group said:

With respect to "Mathematics for mathematics," long-range goals were set forth during the 1963 Cambridge Conference on School Mathematics. Those goals can be accepted in the present report within the framework of the combined science-mathematics curriculum with two major qualifications. The first of these is that the mathematics should, especially during the early elementary school years, draw largely upon the science context both for its motivation and for the concrete situations that it models. This is not contrary to the 1963 report, for the earlier conferees could not assume the relevance, or even the existence, of the science content. Secondly, now that many new objectives have been added, the 1963 "goals" must for the time being be thought of as ideal objectives, to be attained only to the extent that the learning process can be accelerated by the integrated units and the pedagogical methods suggested in this report.

References

[1] Johnson, D. and Rising, G.R. Guidelines for Teaching Mathematics. Belmont, California: Wadsworth Publishing Company, Inc., 1967, p. 11.

[3] Goals for the Correlation of Elementary Science and Mathematics. The Report of the Cambridge Conference on the Correlation of Science and Mathematics. Boston, Massachusetts: Houghton Mifflin Company, 1969.

[2] Husen, T., ed. International Study of Achievement in Mathematics---A Comparison of Twelve Countries Volume 1. New York: John Wiley & Sons, 1967.

OVERT and COVERT (Affective)

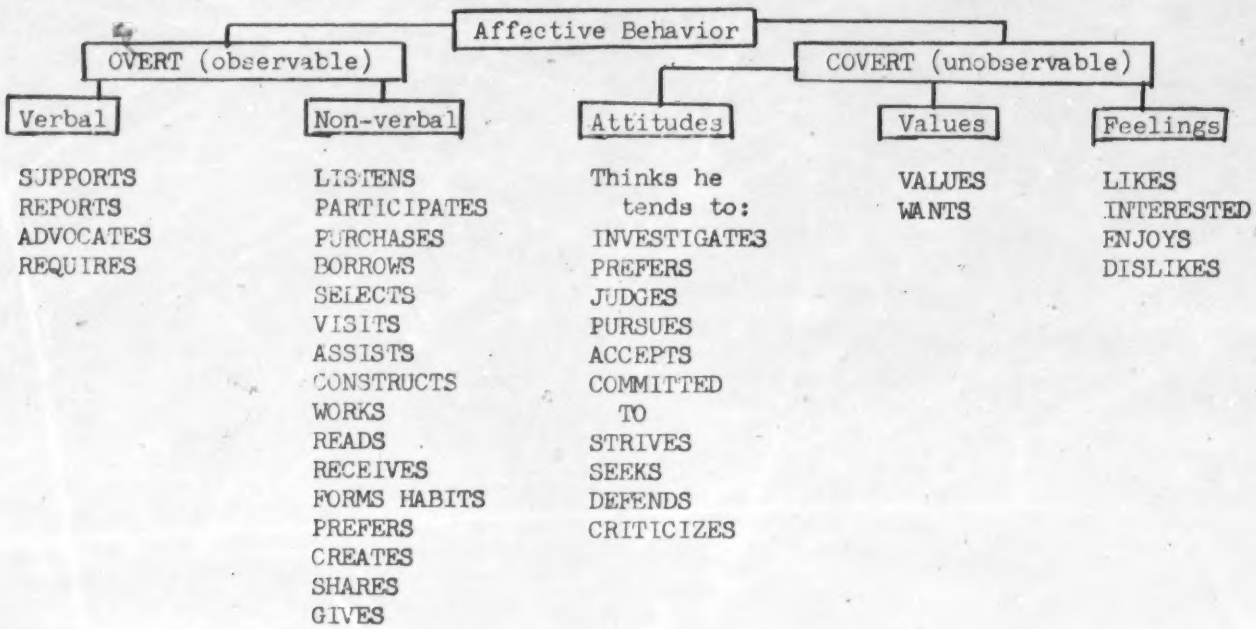
Learner Behaviors in Math **

Some Affective Behaviors of learners can be observed (OVERT), while some are internalized and cannot be seen(COVERT). COVERT behaviors can be assessed by indirectly asking the learner his values, satisfactions, appreciations, attitudes, etc, about what is being learned.

Some types of behavior and some methods of determining their attainment are:

Types of Behavior:	Measured by:
1. OVERT:	
a. Verbal	a. General observation using tape recorder and such techniques as Interaction Analysis.
b. Non-verbal	b. Observational rating scales scored by the teacher.
2. COVERT:	
a. Attitudes	a. Self-evaluation inventories; support scales.
b. Feelings	b. Likert scales; semantic differential scales; conversations.
c. Values	c. Same as "b" above.

The relationship of the OVERT and COVERT Behaviors of the Affective Domain are shown below: (Note: ACTION VERBS as applied to math are shown in CAPS.)



** Sund and Picard: Behavioral Objectives and Evaluation Measures: Science & Math, Merrill, 1972.

The Behavior - Content Grid: A Technique for Analyzing and Evaluating Objectives.*

A Behavior - Content Grid is a device for visualizing relationships between content and behavior. It may be used in selection, formulation, and evaluation of behavioral objectives. It is also used in constructing tests, measuring, and reporting achievement. The example below has been constructed for a course in geometry. It may be refined by including sub-categories of content and behavior.

	BEHAVIORS:					
CONTENT:	Knowledge of definitions & terms	Skills and techniques in Manipulation and computation	Translation and Interpretation	Analysis - Select relevant data	Follow and construct proofs	Verify and Criticize
Non-metric Plane Geometry	X					
Measurement of Length and angle	X	X				
Simple closed curves	X					
Circles	X		X	X		
Construction	X	X				
Congruent sets of points	X					
Area of plane Regions	X	X	X			
Similar Figures						
Graphing Points in a Plane	X	X				

* Excerpt from: Sund and Picard: Behavioral objective and Evaluation Measures: Science and Mathematics, Merrill, 1972, pp. 45-46.

The Behavior - Content Grid: A Technique for Analyzing and Evaluating Objectives

Constructing a behavior - content grid:

1. Make a list of the topics to be covered.
2. List the behaviors you think are important.
3. Arrange the content along one side of a rectangular grid and the behaviors along the other.
4. The intersection of a row and a column is called a cell.
5. A check in one of these cells represents an objective involving a particular behavior with a specific piece content. Factual content, for example, usually requires only low level cognitive behaviors, such as, recall or comprehension.

Analyzing objectives using the behavior - content grid:

1. Prepare the lists of behaviors and contents which match the objectives you are analyzing.
2. Locate each objective in one of the cells of the grid.
3. Look at the distribution of the checks on the grid.
   a. Are they evenly distributed or clustered?
   b. Are there many in one column and few in the other?
   c. Do any behaviors have no objectives associated with them?
   d. Does any content area have no objectives?

Look at the sample Behavior - Content Grid. Notice the heavy emphasis on Knowledge of definitions and terms and the lack of objectives requiring higher cognitive behaviors. This is a common situation for initial attempts at writing objectives because low level objectives are easier to write. This situation is also common in traditional programs. Notice in the sample grid that several objectives on circles have been included but none on similar figures. Analysis of a test in this way reveals a preference for certain topics. It establishes the content considered important and unimportant by the test writer.

Use of behavior - content grid:

Try the following:

1. Write a set of math objectives or select a previously written set from some available source. Check to verify that each objective contains the 3 basic criteria for a behavioral objective: learner terminal behavior, conditions, minimal level.
2. Design the content and behavior lists on a behavior - content grid.
3. Analyze the set of objectives with the behavior - content grid.

The Behavior - Content Grid:
A Technique for Analyzing and
Evaluating Objectives

Use of behavior - content grid: (Cont'd)

4. Evaluate the set of objectives by answering:

   a. Haveyou neglected any topics or behaviors?

   b. Have you placed too much emphasis on memory and recall?

   c. Are additional objectives required?

   d. What would this analysis reveal to you about your teaching if the objectives were used as is?

   e. Try designing a set of objectives which show a balance among the cognitive domain levels and then teach and evaluate your teaching to determine how the Behavior - Content Grid helped you.

Some blank Behavior - Content Grids are attached for your use.